Part 2

COLLINS MEASUREMENTS @ BELLE

Phenix SpinFest, Urbana, Illinois
July 2014
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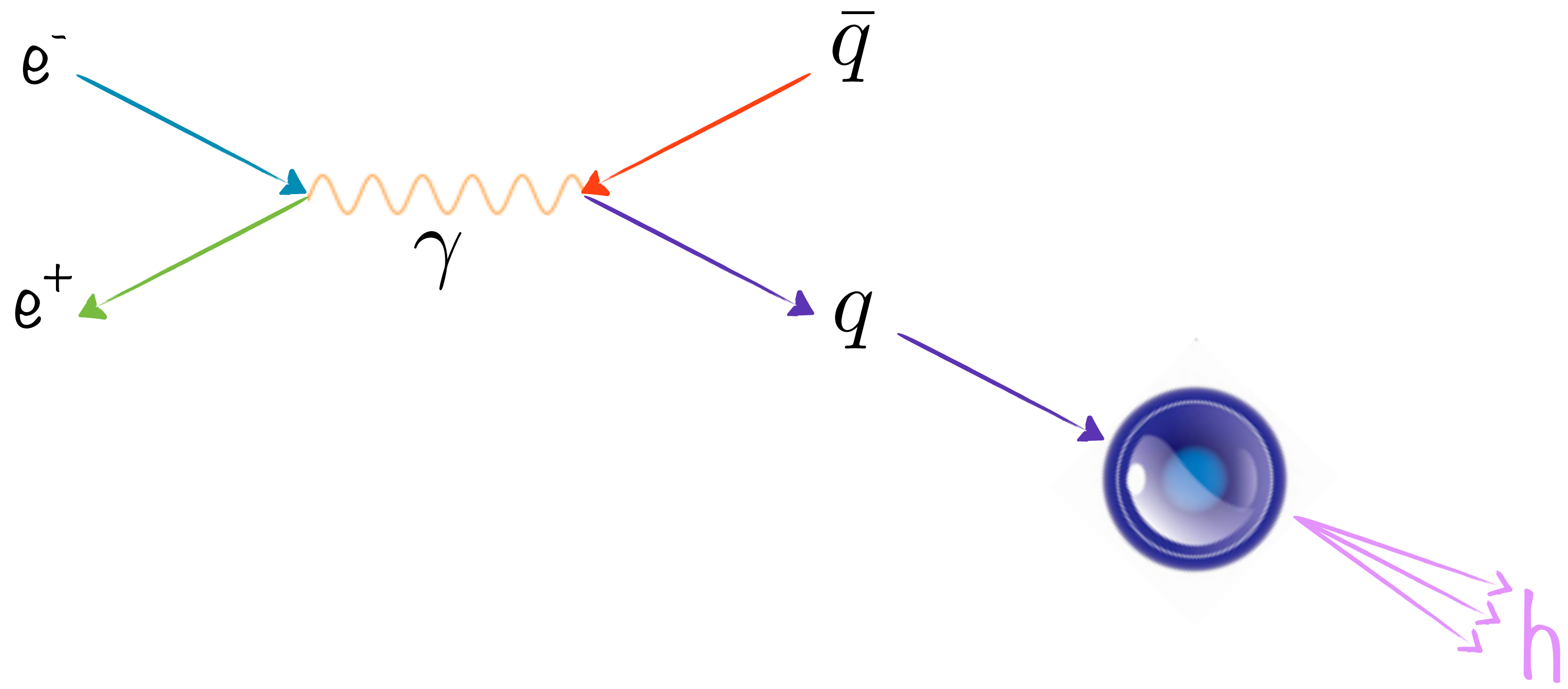
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

BELLE

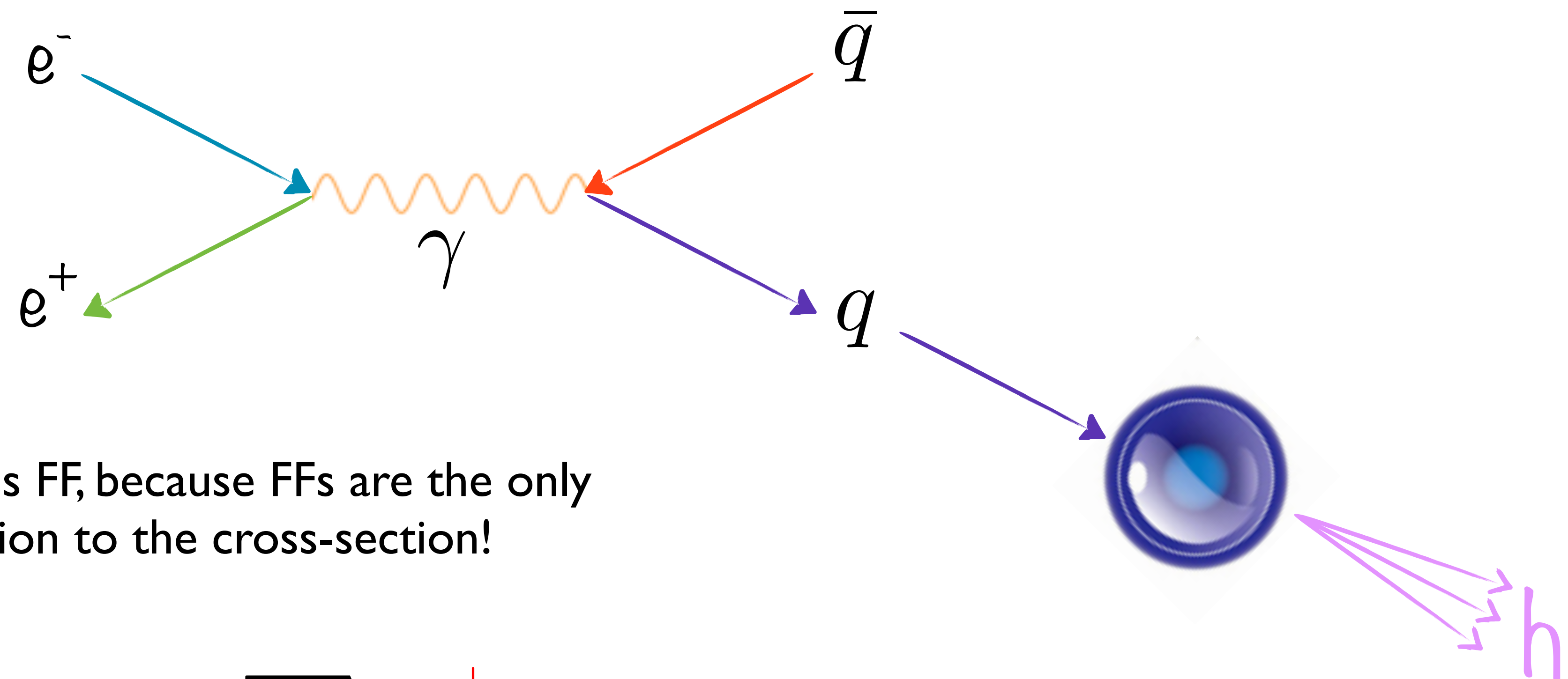
e⁺e⁻ annihilation

$$e^+e^- \to q\,\bar{q} \to hX$$



e^+e^- annihilation

$$e^+e^- \to q\,\bar{q} \to hX$$



e^+e^- is the cleanest way to access FF, because FFs are the only non-perturbative contribution to the cross-section!

$$\sigma^{e^+e^- \to hX} \propto \sum_q \sigma^{e^+e^- \to q\bar{q}} \times (D_q^h + D_{\bar{q}}^h)$$

e^+e^- annihilation

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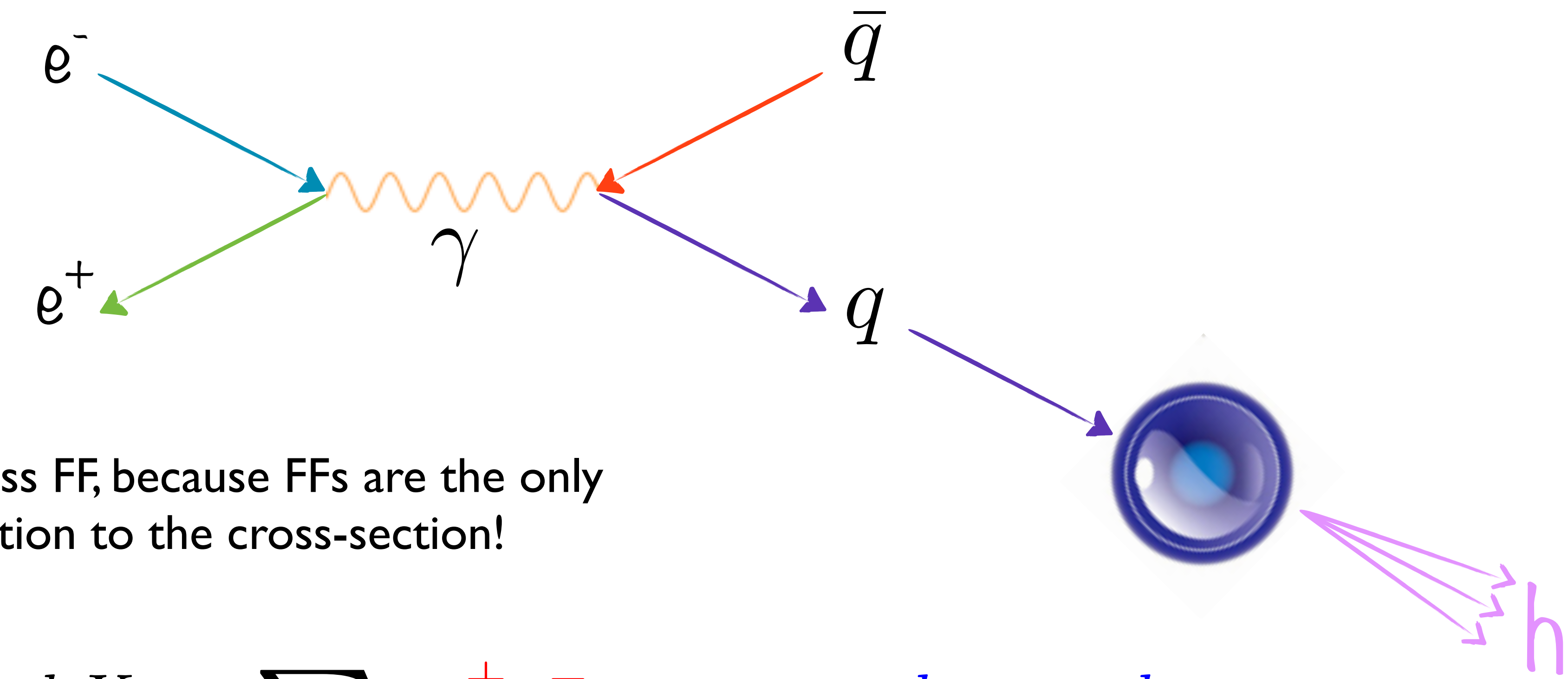

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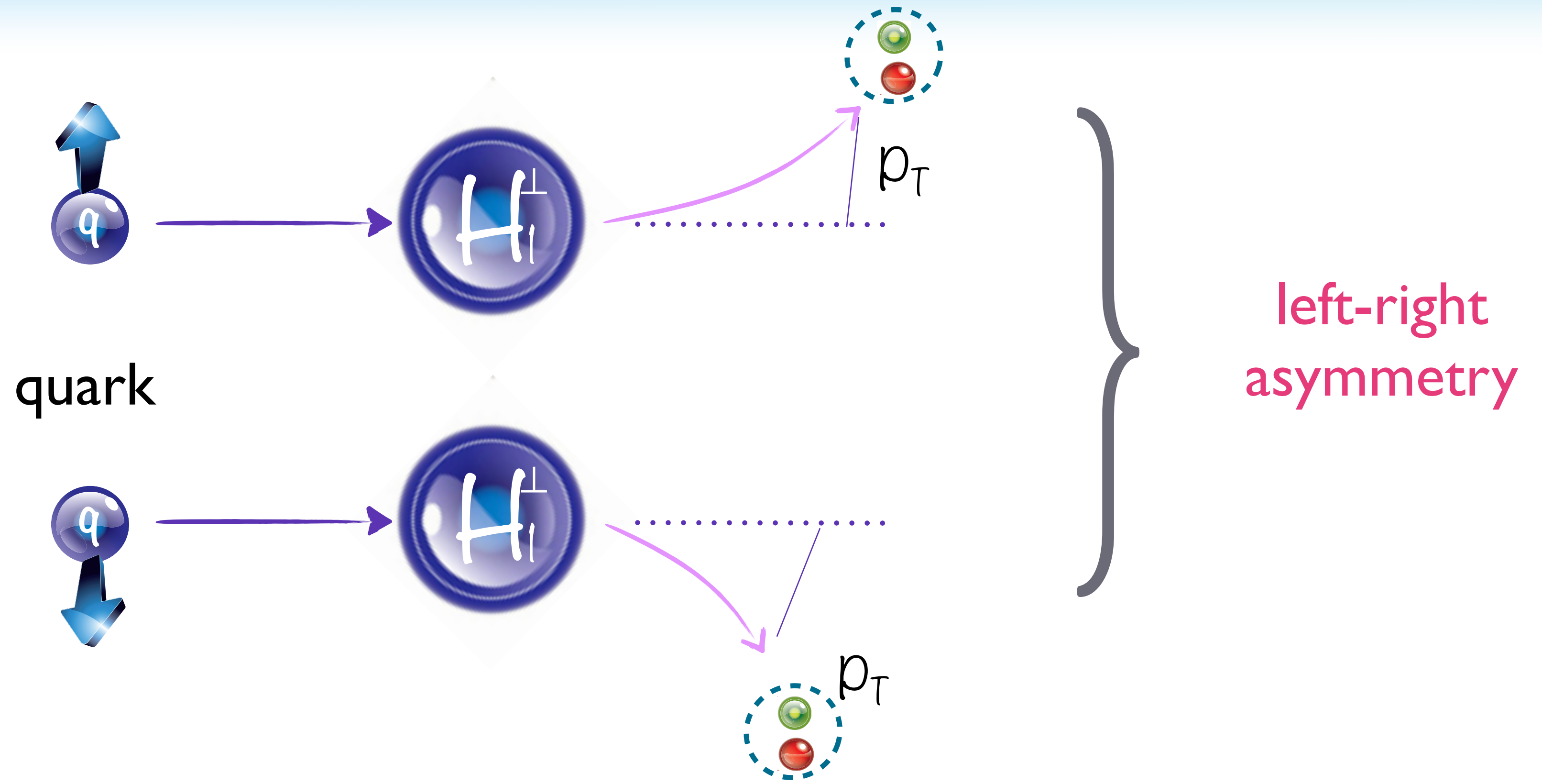
$$\sigma^{e^+e^- \to hX} \propto \sum_q \sigma^{e^+e^- \to q\bar{q}} \times (D_q^h + D_{\bar{q}}^h)$$

$D_q^h(z)$ is the probability that an hadron h with energy z is generated from a parton q

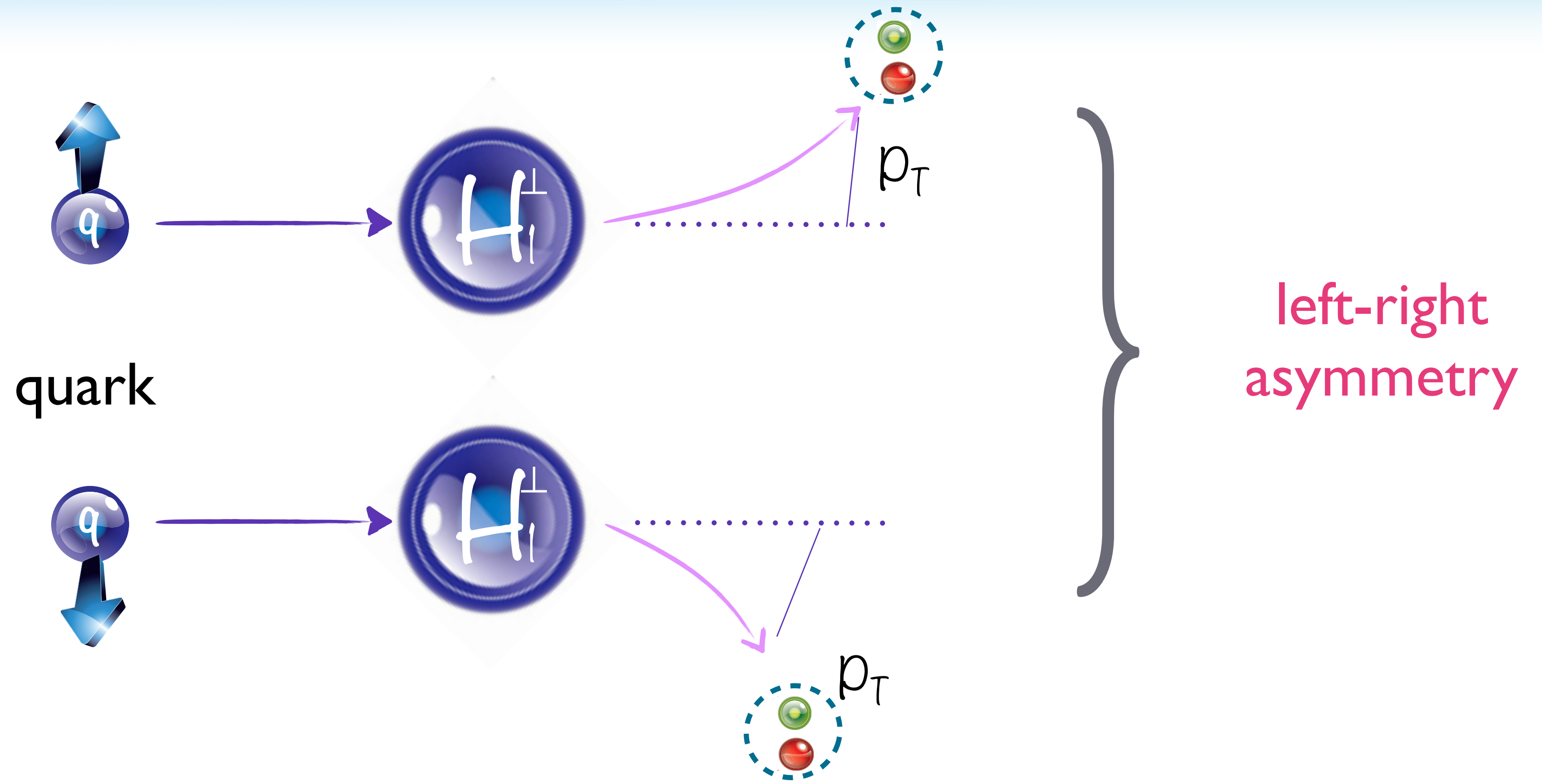
$$z \equiv \frac{E_h}{E_q} = \frac{E_h}{E_b} = \frac{2E_h}{Q}$$

Collins Fragmentation



Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

Collins Fragmentation

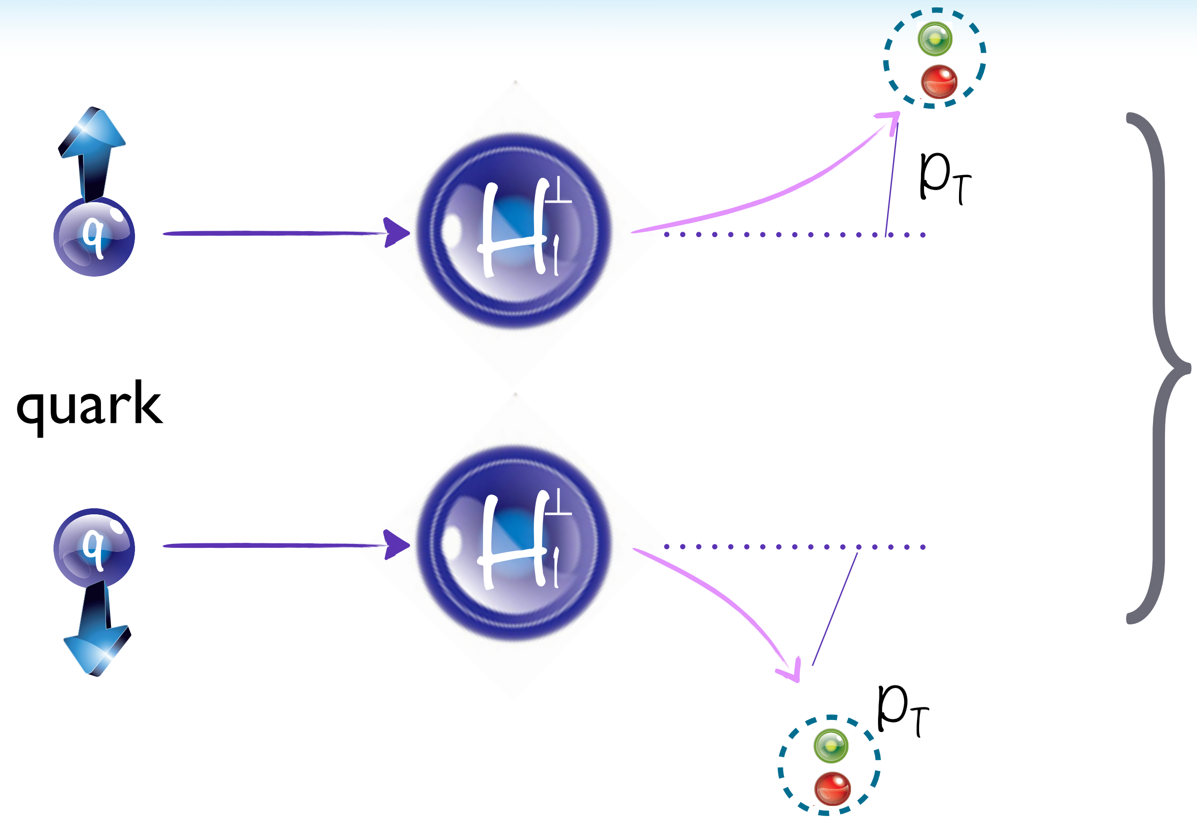


Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

strictly related to the outgoing hadron tranverse momentum

TMD!

Collins Fragmentation



left-right asymmetry

Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

strictly related to the outgoing hadron tranverse momentum

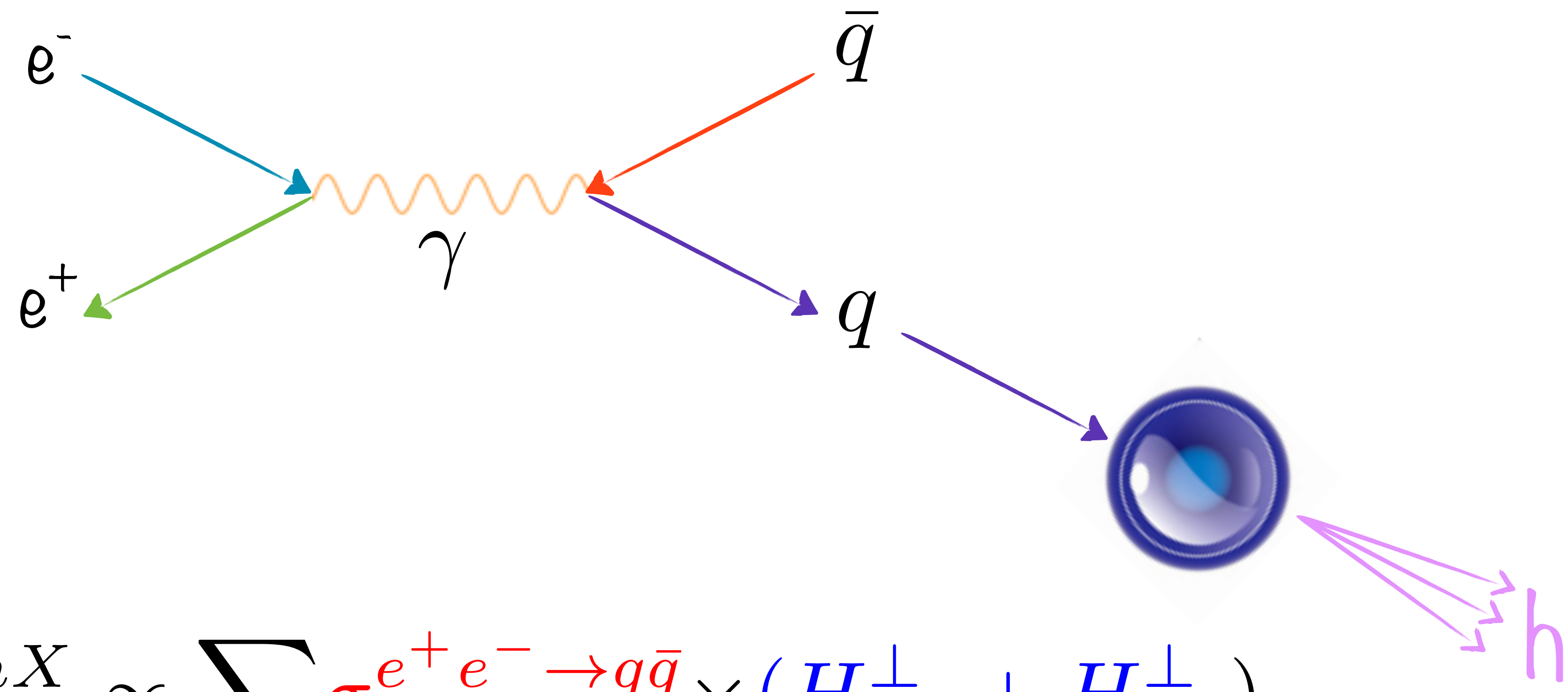
TMD!

Chiral odd!

$$\underbrace{\underset{\text{chiral odd}}{X} \otimes \underset{\text{chiral odd}}{H_1^\perp}}_{\text{chiral even}}$$

Collins Fragmentation

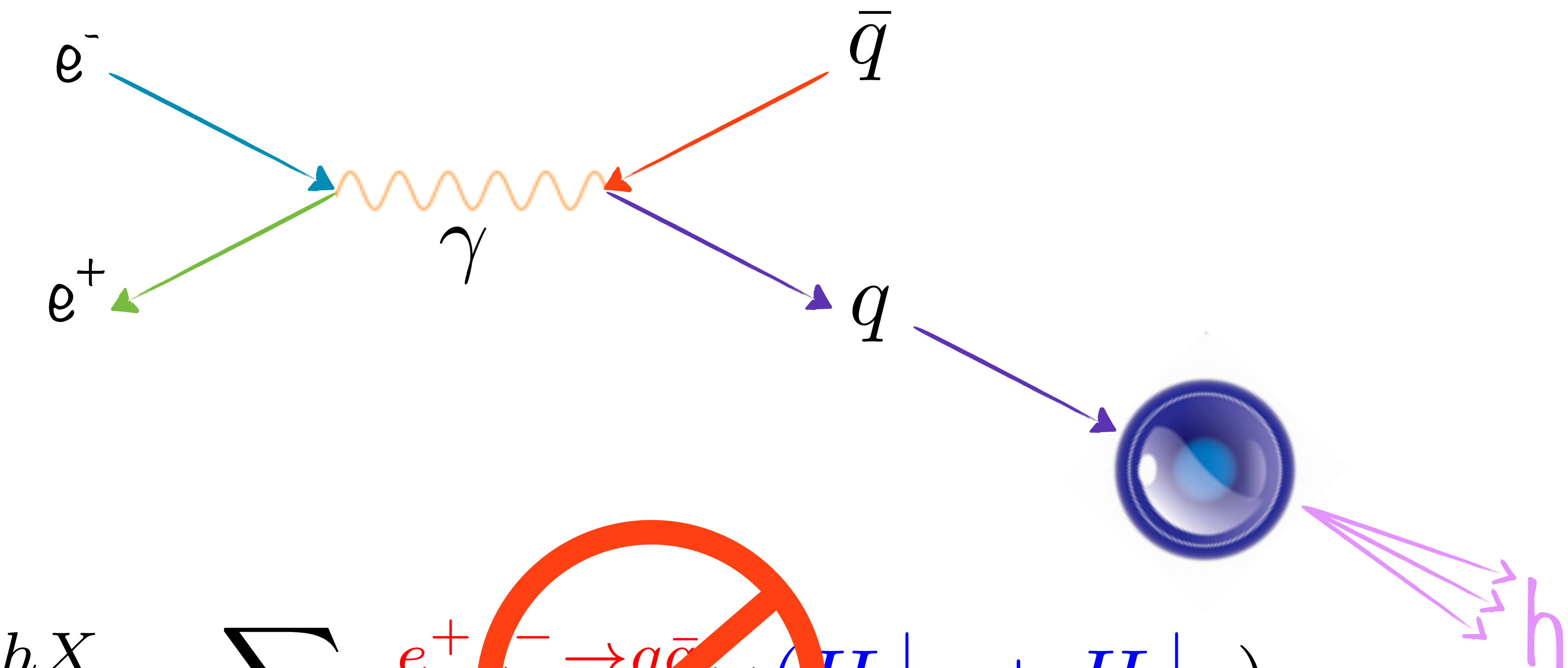
$$e^+e^- \rightarrow q\,\bar{q} \rightarrow hX$$



$$\sigma^{e^+e^-\rightarrow hX} \propto \sum_q \sigma^{e^+e^-\rightarrow q\bar{q}} \times (H_{1,q}^{\perp} + H_{1,\bar{q}}^{\perp})$$

Collins Fragmentation

$$e^+e^- \to q\,\bar{q} \to hX$$



$$\sigma^{e^+e^- \to hX} \propto \sum_q \sigma^{e^+e^- \to q\bar{q}} \times \left(H_{1,q}^{\perp} + H_{1,\bar{q}}^{\perp}\right)$$

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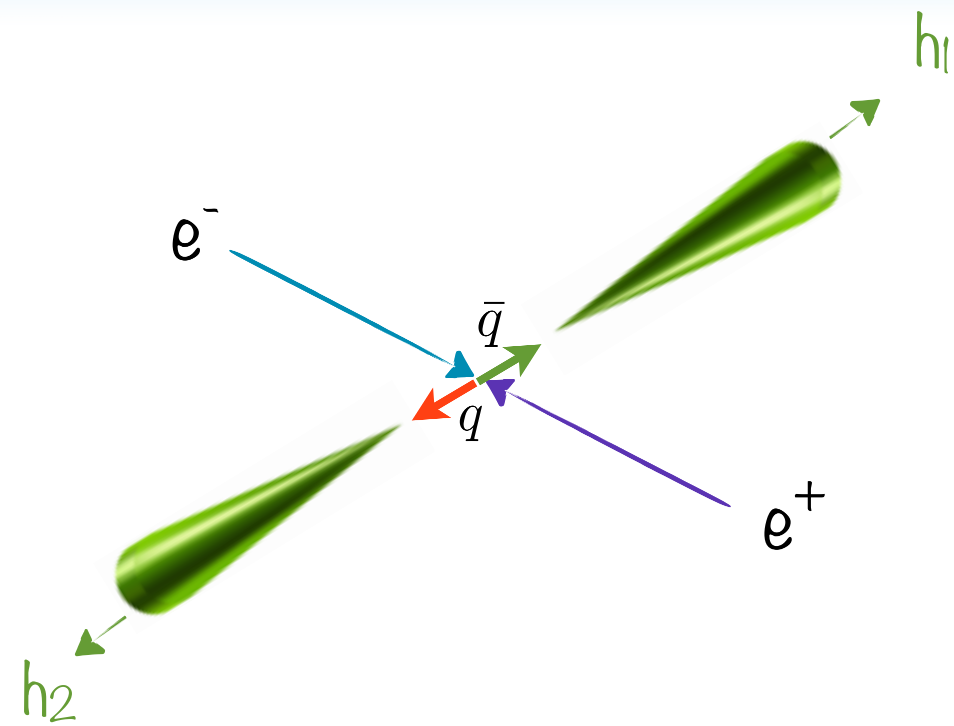
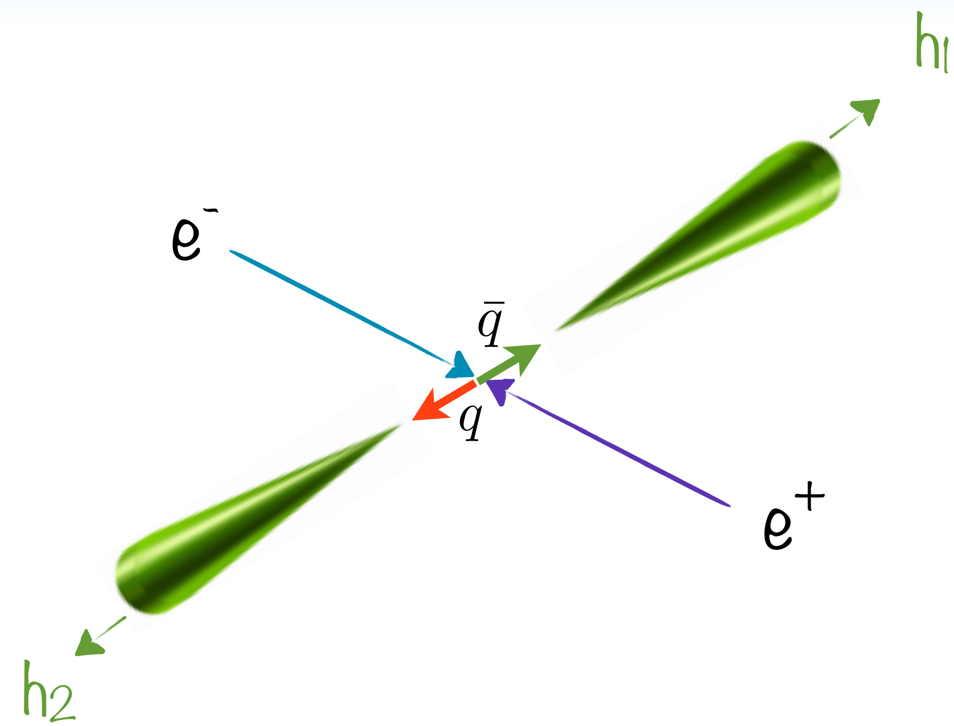
Collins Fragmentation



Back-to-Back jets

In e+e- reaction, there is no fixed transverse axis to define azimuthal angles to, and even if there were one the net quark polarization would be 0

Collins Fragmentation



Back-to-Back jets

In e+e- reaction, there is no fixed transverse axis to define azimuthal angles to, and even if there were one the net quark polarization would be 0

But if we look at the whole event, even though the q and q̄ spin directions are unknown, they must be parallel

$$e^+e^- \to q\,\bar{q} \to h_1\,h_2\,X$$

$$h = \pi,\ K$$

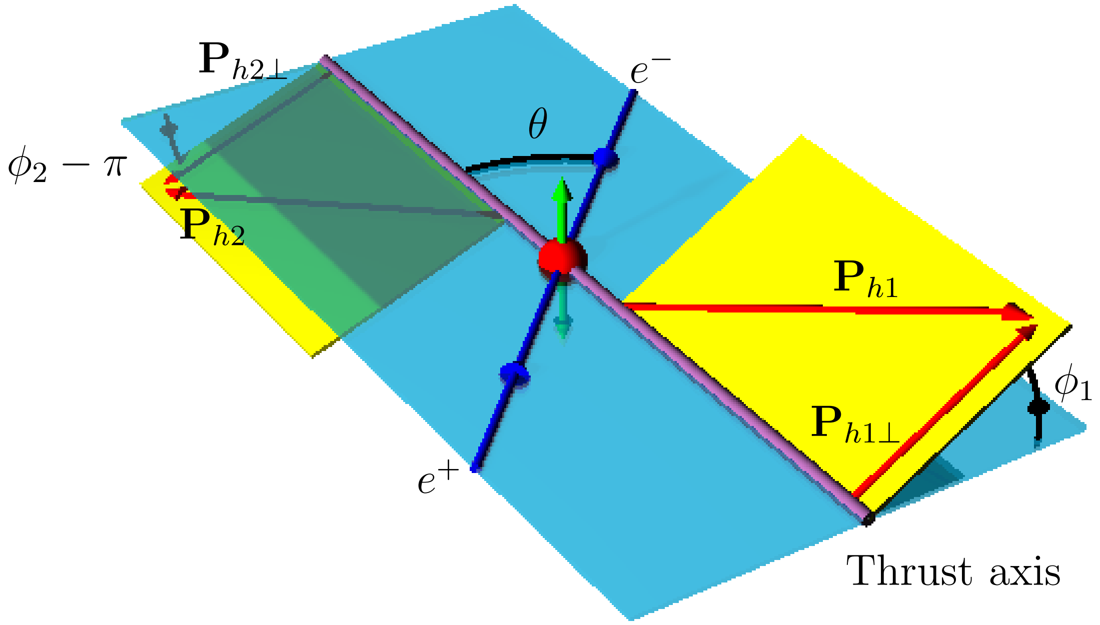
Reference frames

$$e^+e^- \to q\,\bar{q} \to h_1\,h_2\,X$$

$$h = \pi,\ K$$

$\phi_1+\phi_2$ method:

hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy

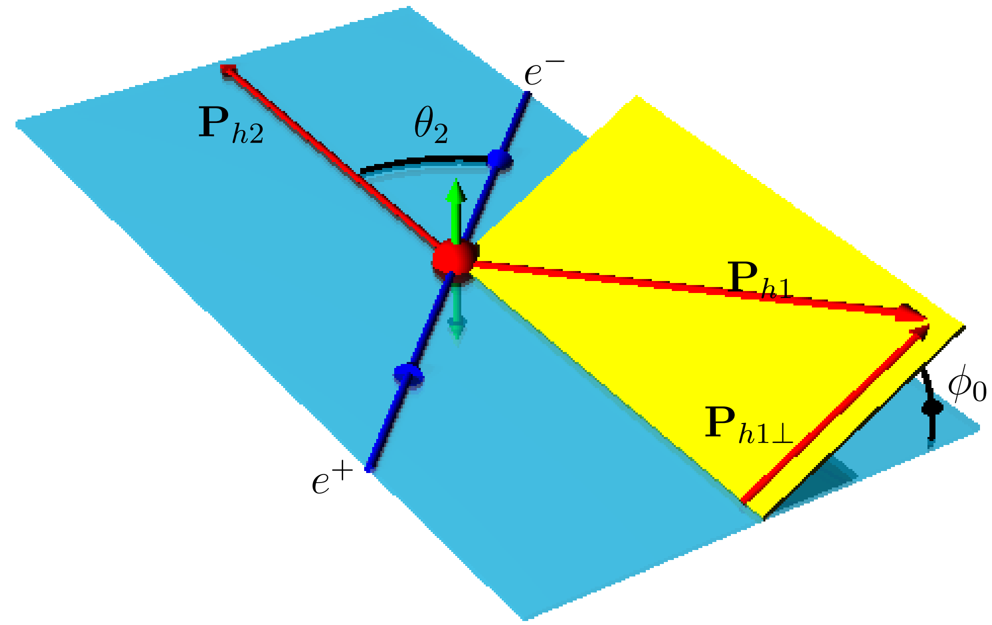


reference plane (in blue) given by the e+e- direction and the $q\bar{q}$ axis

Thrust axis= proxy for the $q\bar{q}$ axis

ϕ_0 method:

hadron 1 azimuthal angle with respect to hadron 2



reference plane (in blue) given by the e+e- direction and one of the hadron

Reference frames

$$e^+e^- \rightarrow q\,\bar{q} \;\rightarrow h_1\,h_2\,X$$

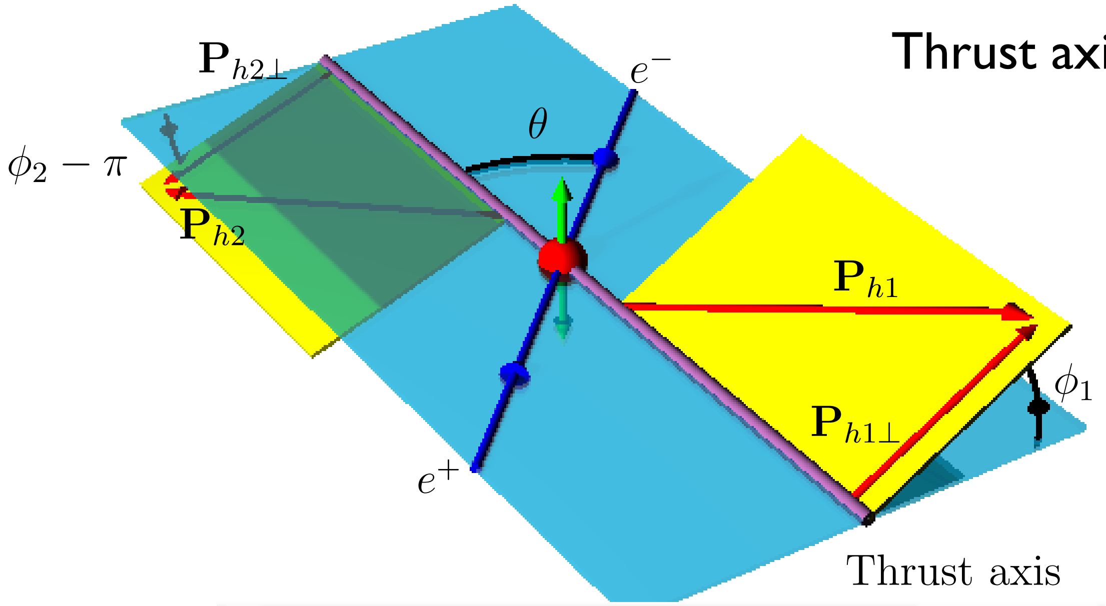
$$h = \pi,\ K$$

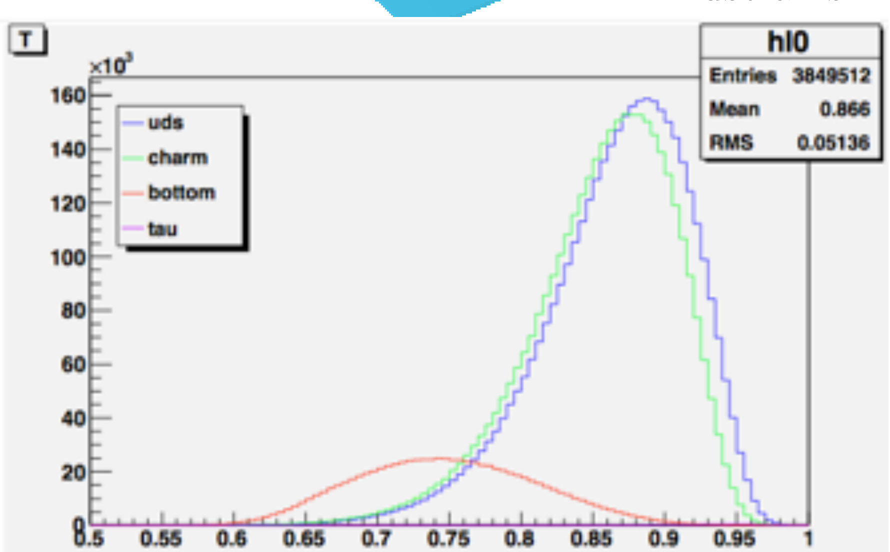
ϕ_1+ϕ_2 method:

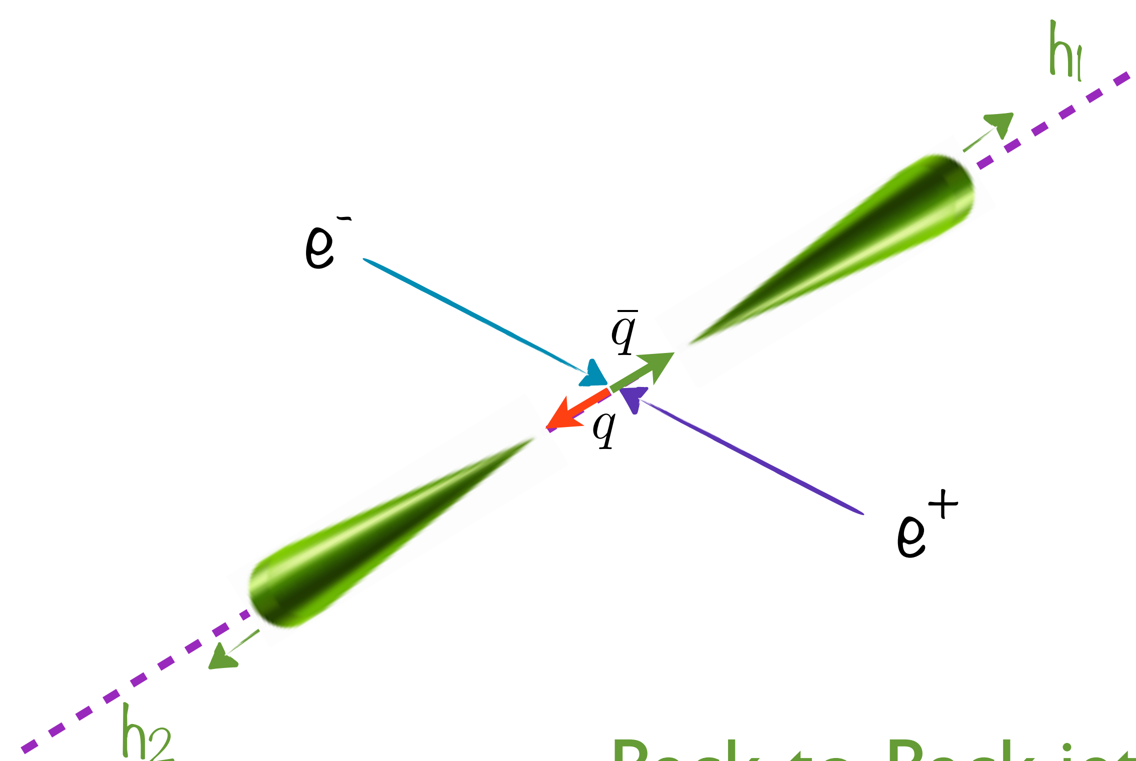
hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy

$$thrust = max\left|\frac{\sum_{i=1}^{N}|(\hat{\mathbf{n}}\cdot\mathbf{P}_i)|}{\sum_{i=1}^{N}|\mathbf{P}_i|}\right|$$

Thrust axis= proxy for the $q\bar{q}$ axis







Back-to-Back jets

Reference frames

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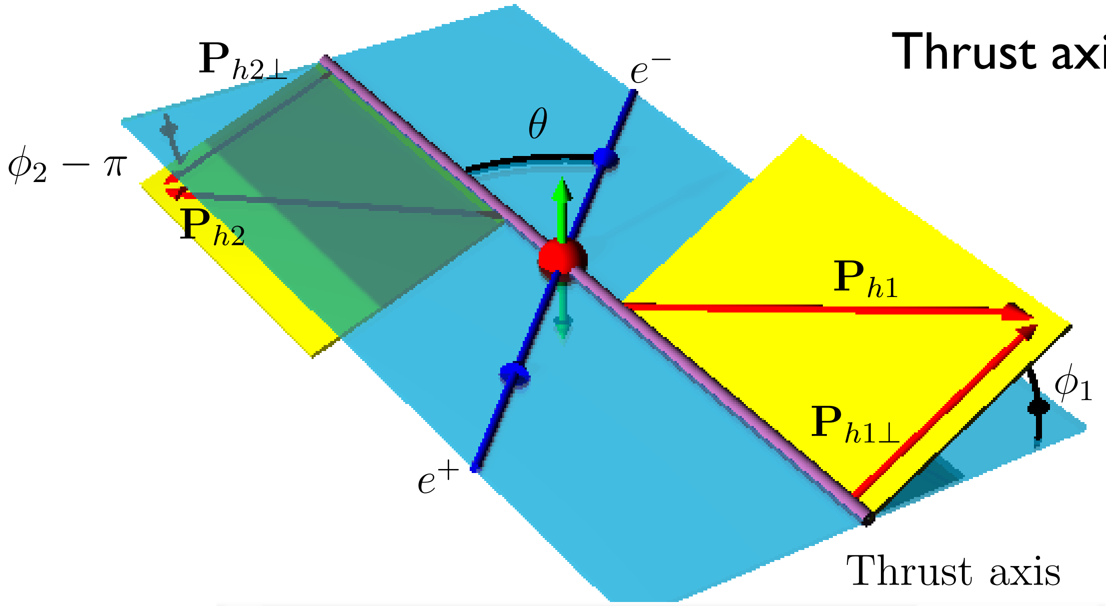

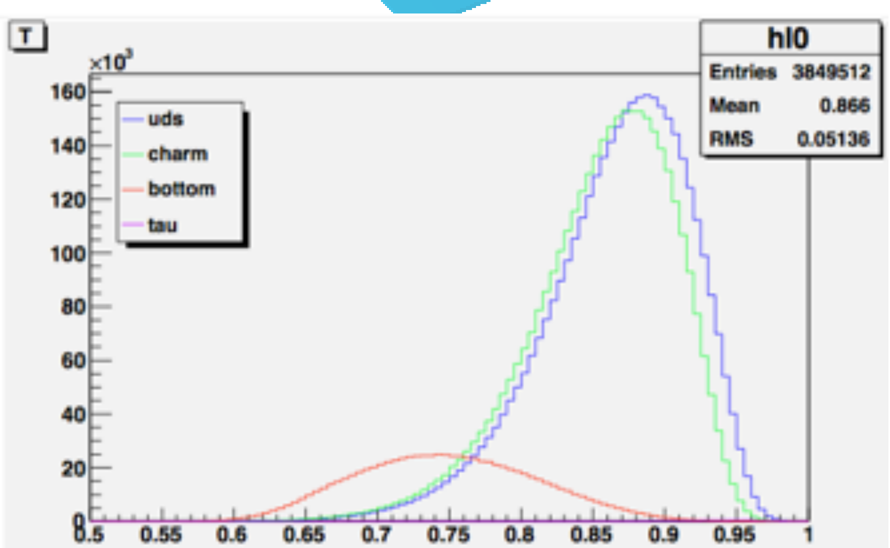

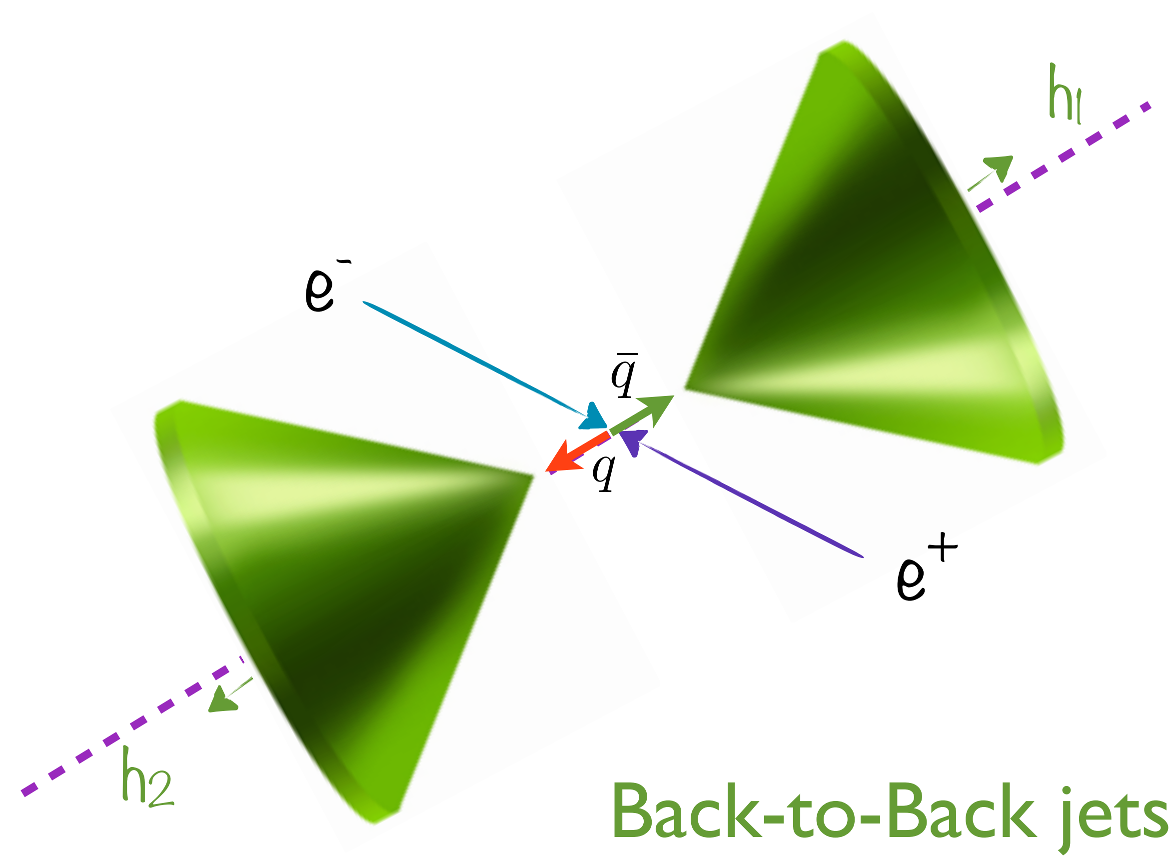

Back-to-Back jets

Reference frames

$$e^+e^- \to q\,\bar{q} \to h_1\,h_2\,X \qquad h = \pi,\ K$$

ϕ_1+ϕ_2 method:

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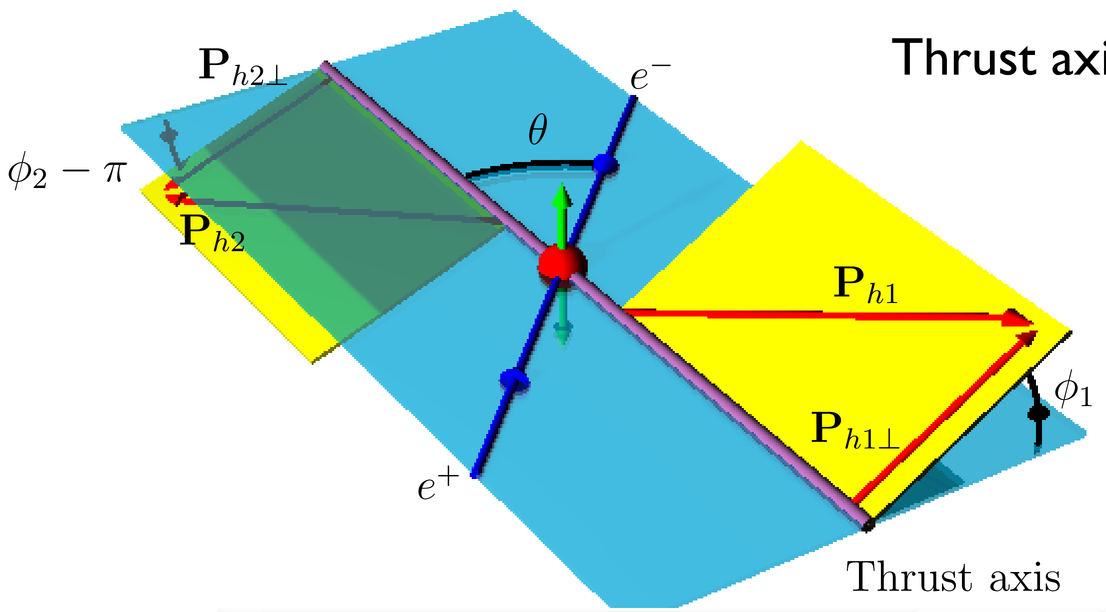
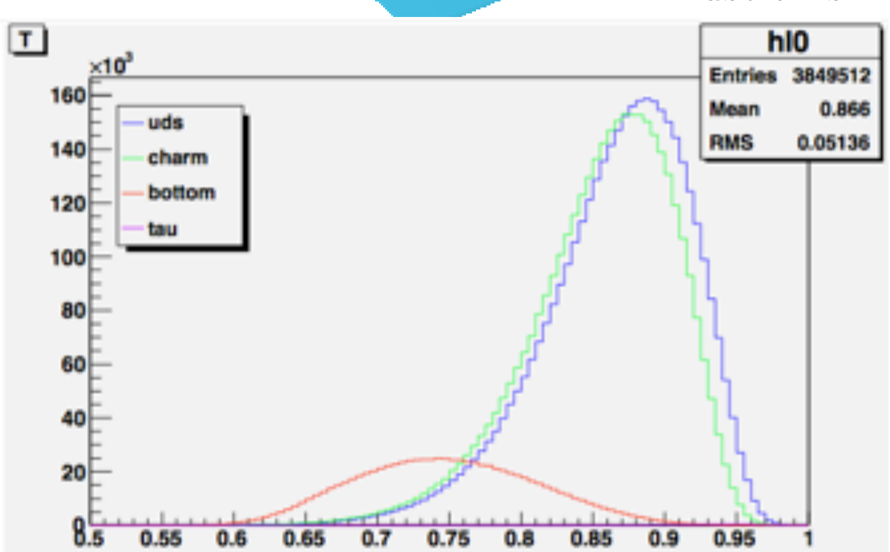
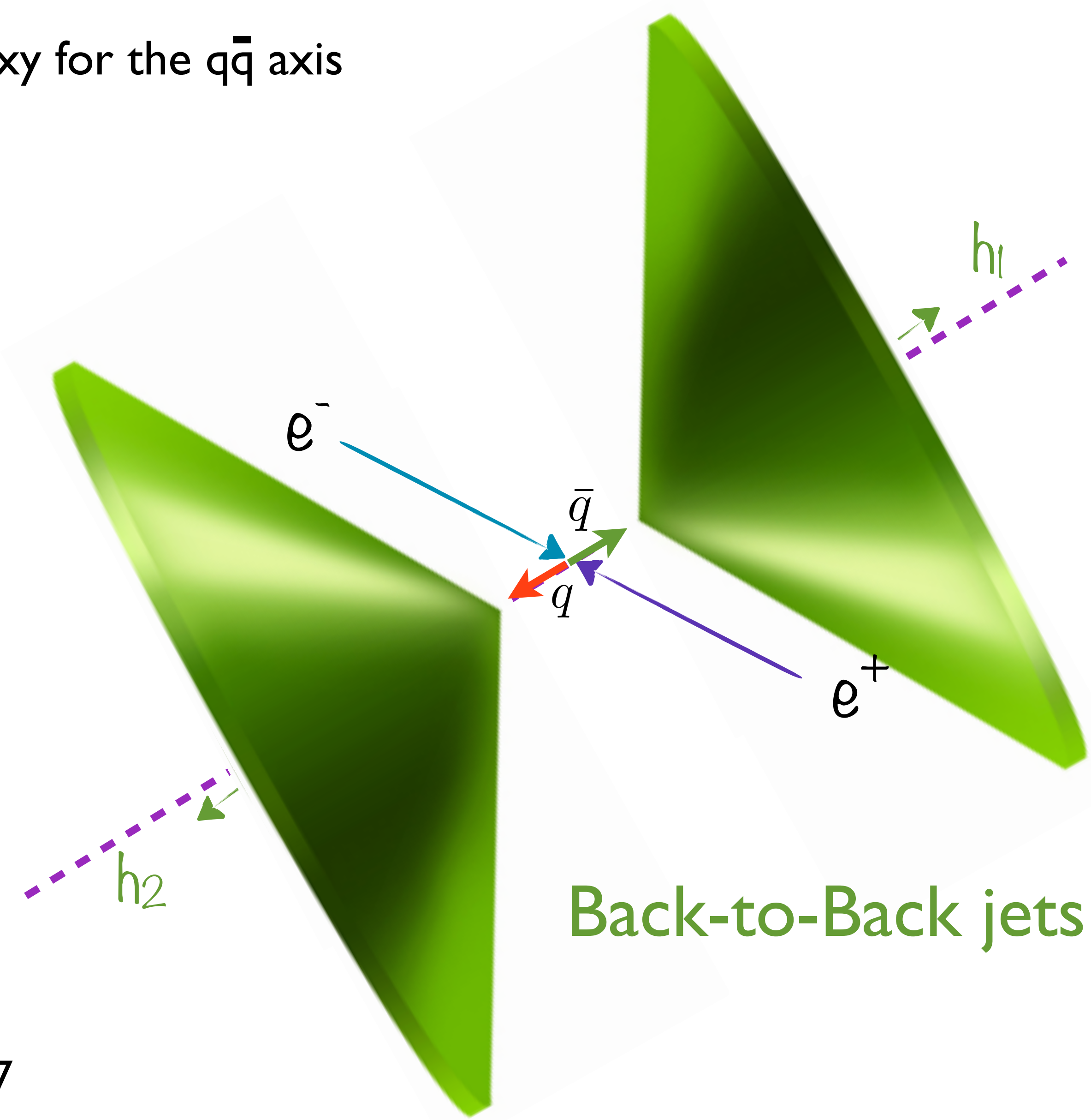


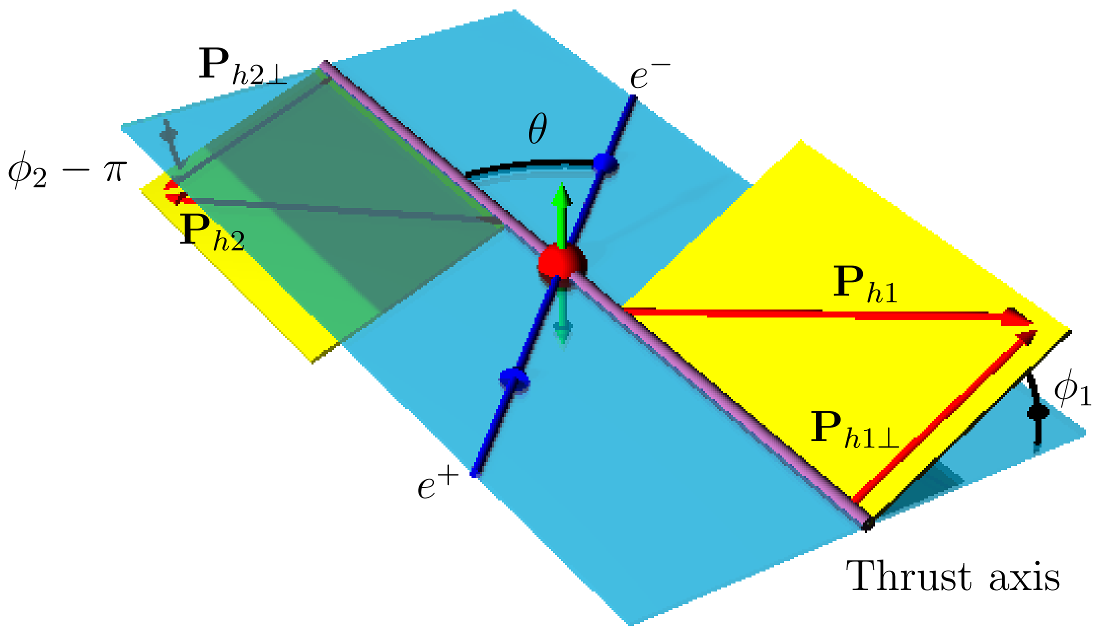




Reference frames

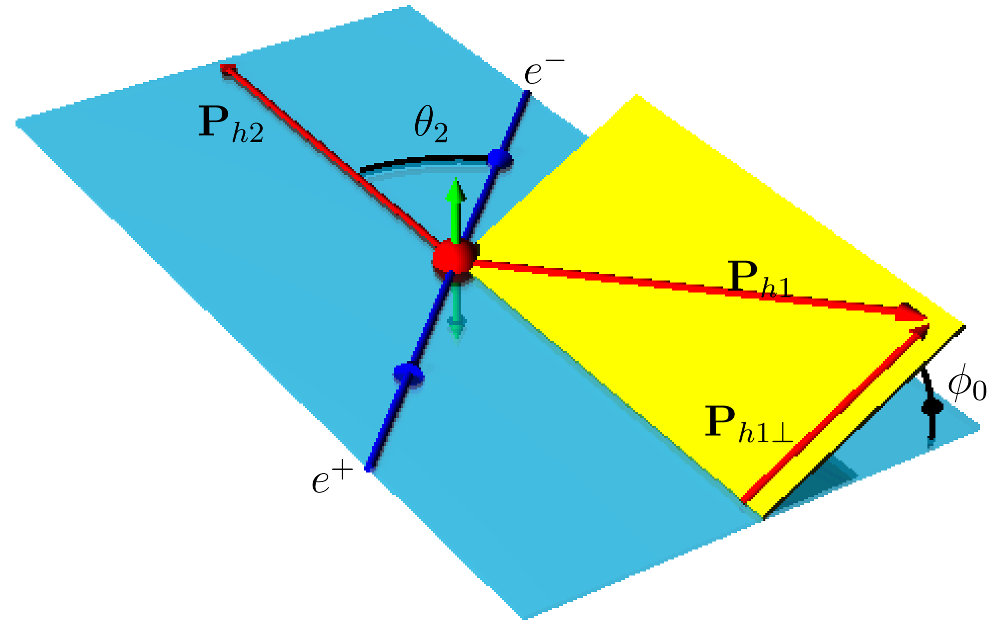
$\phi_1+\phi_2$ method:

hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy



ϕ_0 method:

hadron 1 azimuthal angle with respect to hadron 2



$$\sigma \sim \mathcal{M}_{12}\Big(1+\frac{\sin^2\theta_T}{1+\cos^2\theta_T}\, cos(\phi_1+\phi_2)\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

$$F^{[n]}(z_i) \equiv \int d|k_T|^2\left[\frac{|k_T|}{M_i}\right]^{[n]} F(z_i,|k_T|^2)$$

$$\sigma \sim \mathcal{M}_0\Big(1+\frac{\sin^2\theta_2}{1+\cos^2\theta_2}\, cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$

$$\mathcal{F}[X] = \sum_{q\bar{q}}\int [2\hat{\mathbf{h}}\cdot\mathbf{k_{T1}}\hat{\mathbf{h}}\cdot\mathbf{k_{T2}} - \mathbf{k_{T1}}\cdot\mathbf{k_{T2}}]$$

$$d^2\mathbf{k_{T1}}d^2\mathbf{k_{T2}}\,\delta^2(\mathbf{k_{T1}}+\mathbf{k_{T2}}-\mathbf{q_T})X$$

$$k_{Ti} = z_i\, p_{Ti}$$

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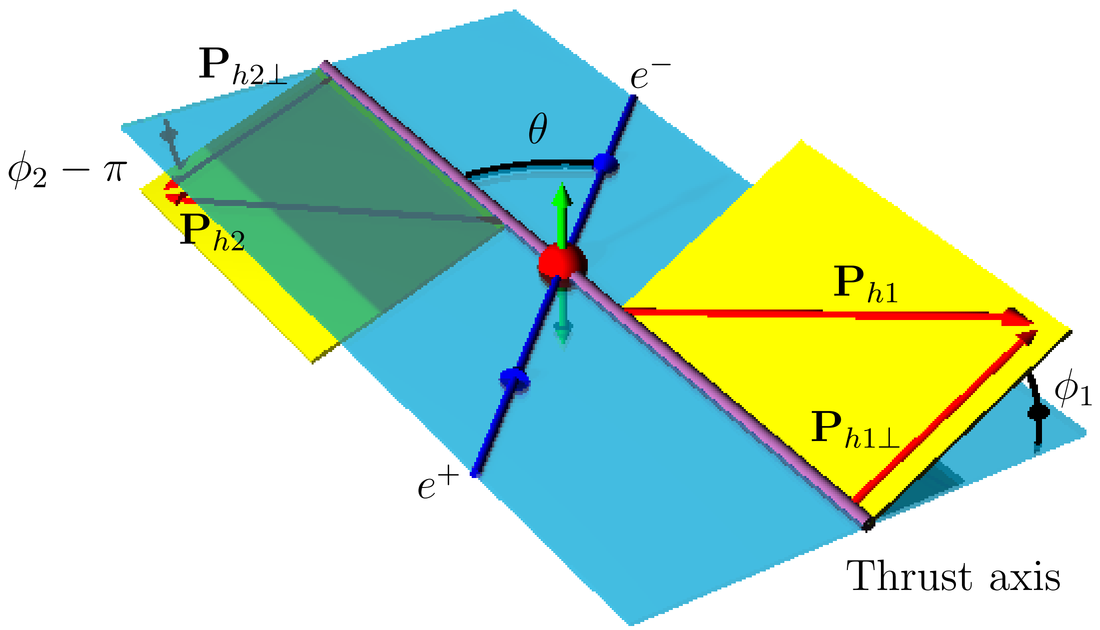
Nucl.Phys.B806:23,2009

Reference frames

$\phi_1+\phi_2$ method:

hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy



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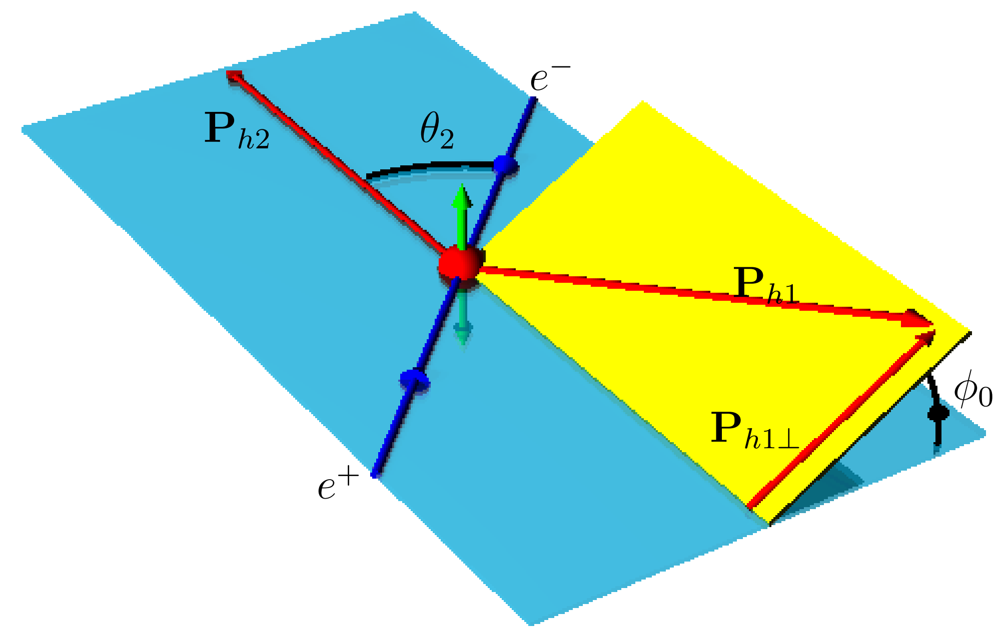
hadron 1 azimuthal angle with respect to hadron 2



$$\sigma \sim \mathcal{M}_{12}\Big(1 + \frac{\sin^2\theta_T}{1+\cos^2\theta_T}\, cos(\phi_1+\phi_2)\boxed{\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}}\Big)$$

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$$k_{Ti} = z_i\, p_{Ti}$$

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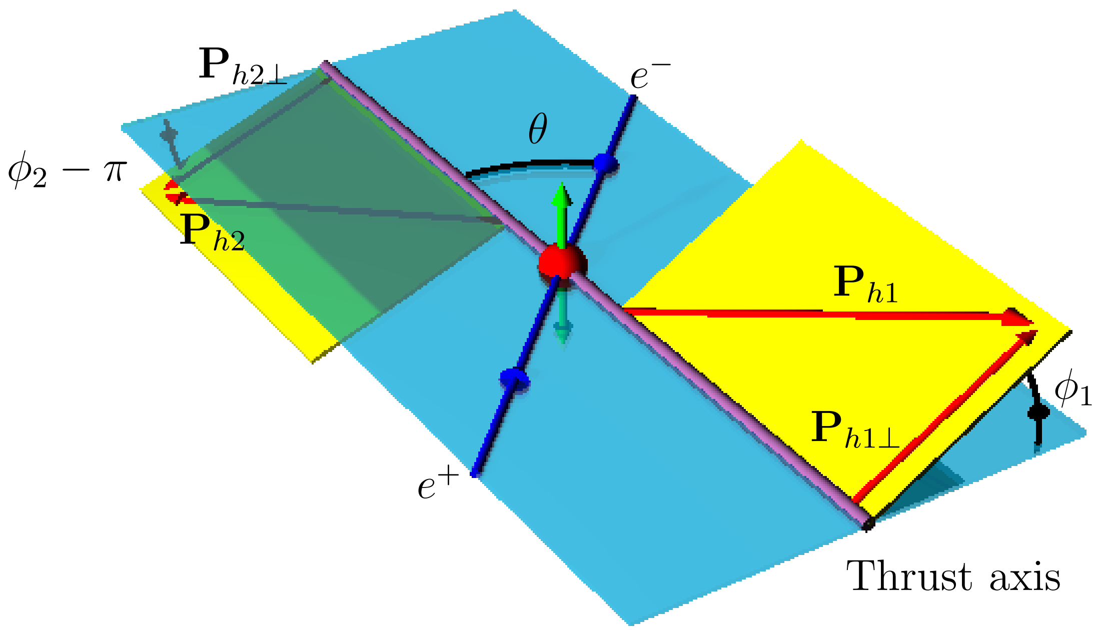
Reference frames

$\phi_1+\phi_2$ method:

hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy



$$\sigma \sim \mathcal{M}_{12}\left(1+\frac{\sin^2\theta_T}{1+\cos^2\theta_T}\, cos(\phi_1+\phi_2)\boxed{\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}}\right)$$

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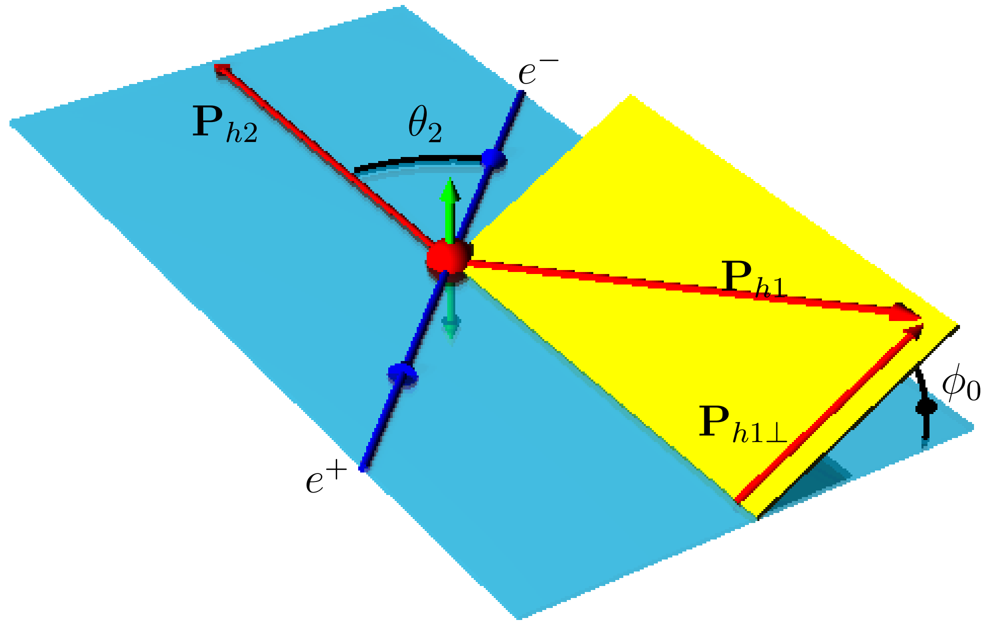

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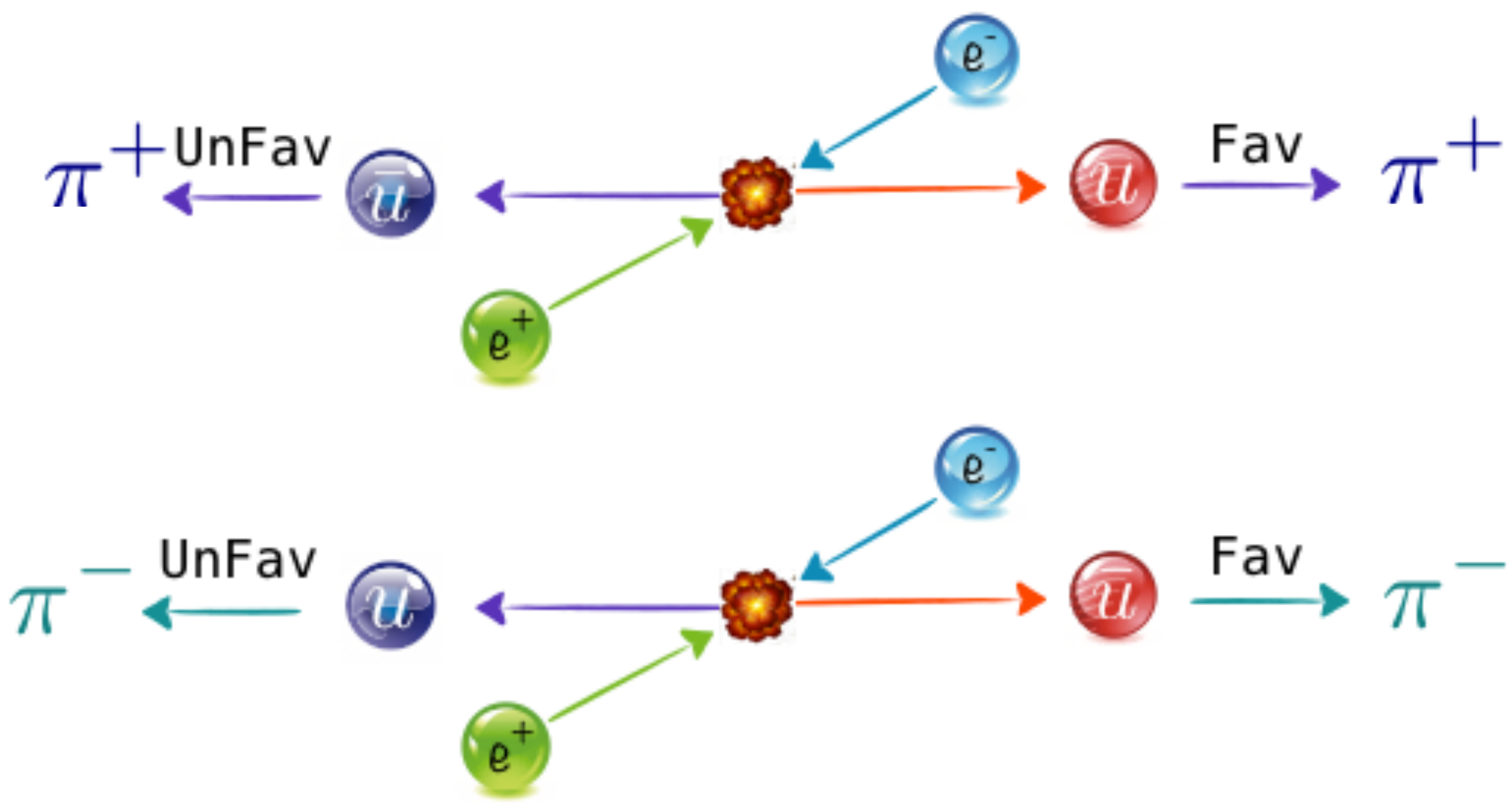
Product of 2 Collins FFs

Like-sign couples



Product of 2 Collins FFs

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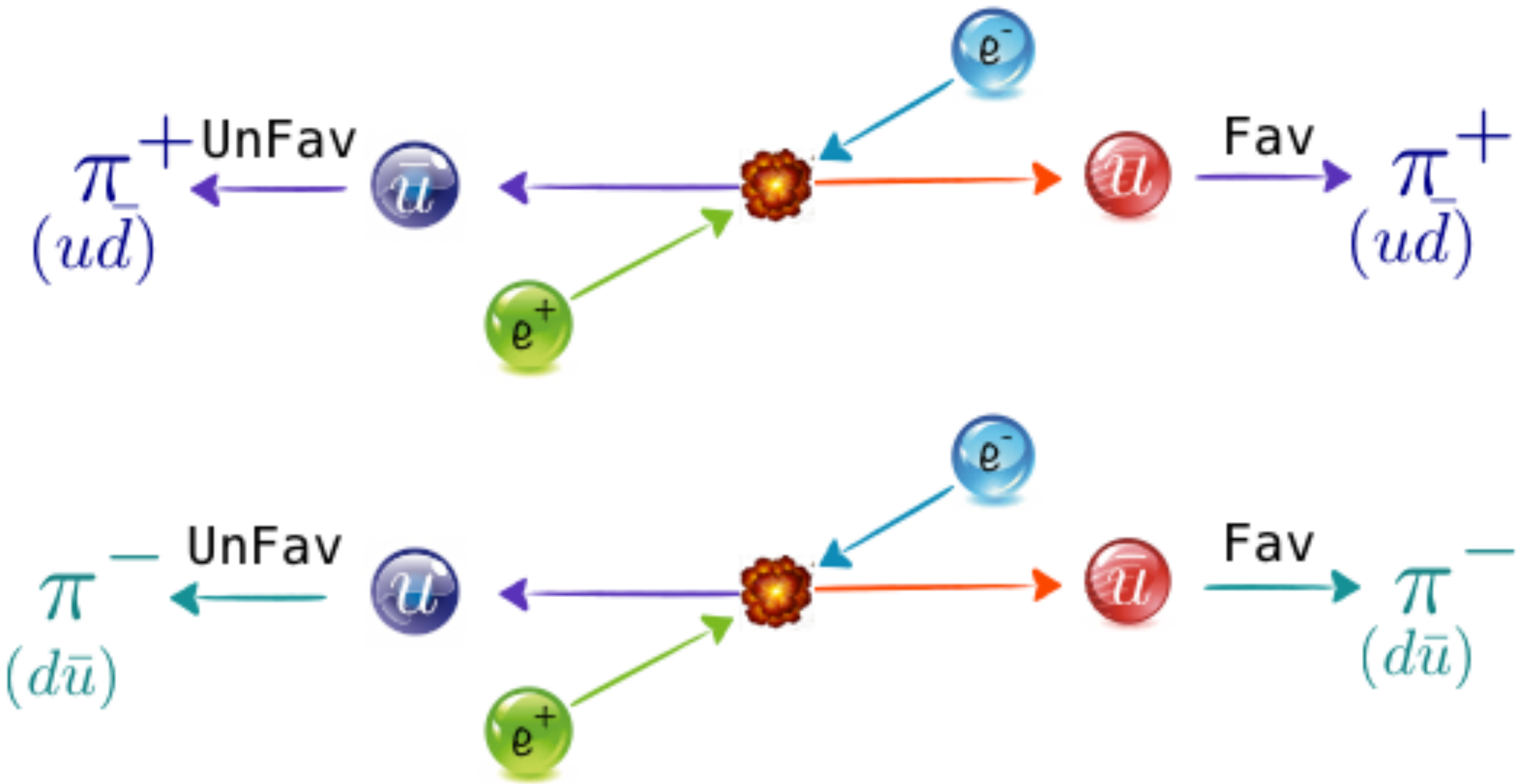

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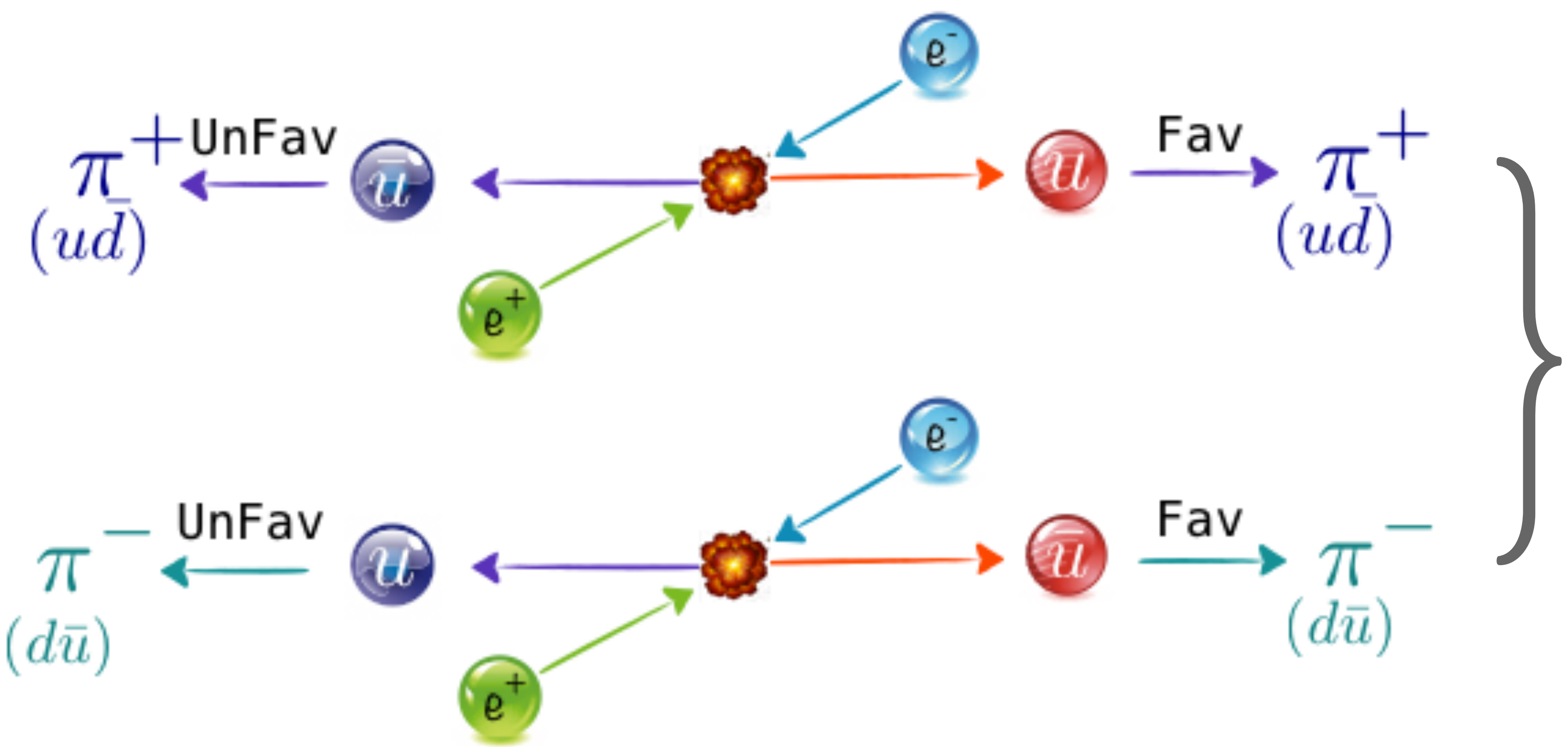
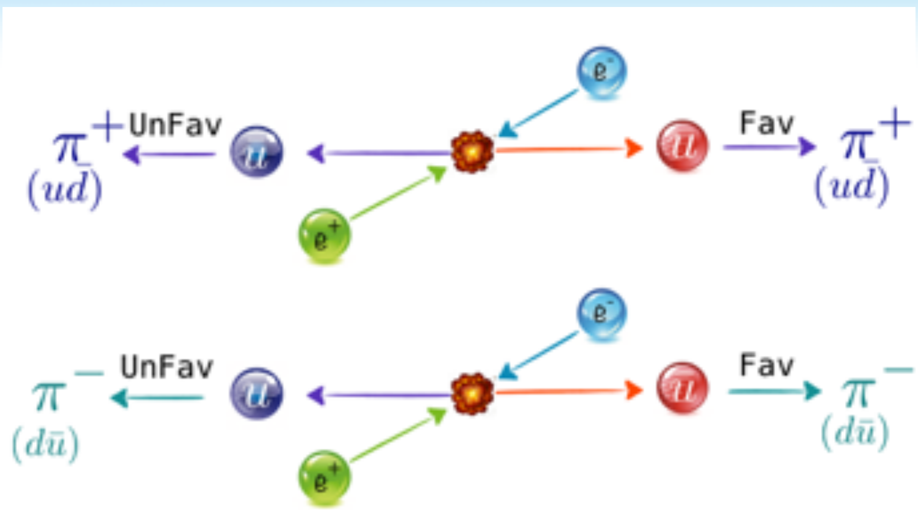


[Fav · UnFav] + [Fav · UnFav]

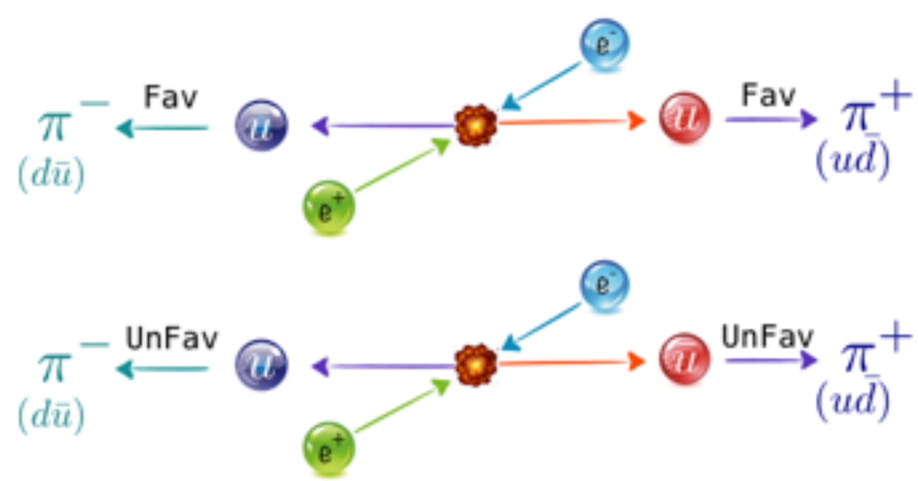
Product of 2 Collins FFs

Like-sign couples



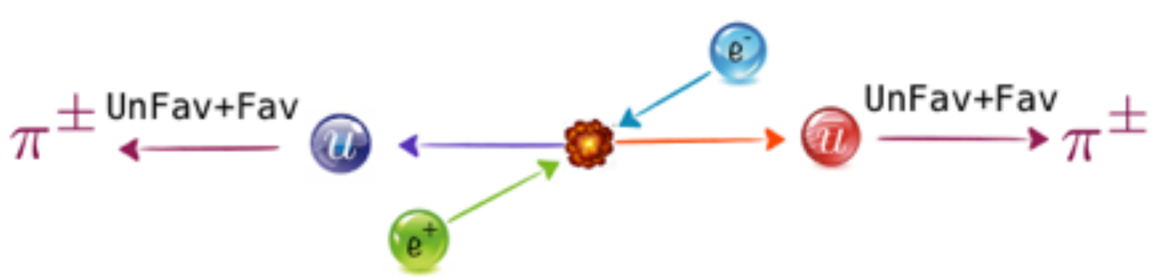
$$[\mathrm{Fav} \cdot \mathrm{UnFav}] + [\mathrm{Fav} \cdot \mathrm{UnFav}]$$

Unlike-sign couples



$$[\mathrm{Fav} \cdot \mathrm{Fav}] + [\mathrm{UnFav} \cdot \mathrm{UnFav}]$$

All charges couples



$$[\mathrm{Fav} + \mathrm{unFav}] \cdot [\mathrm{UnFav} + \mathrm{Fav}]$$

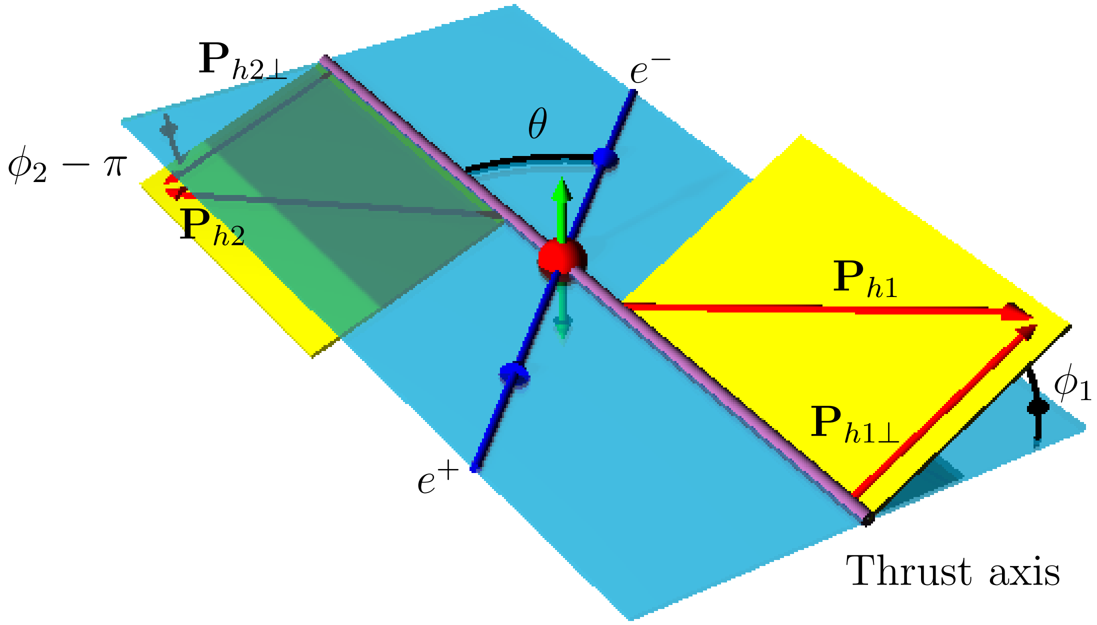
Reference frames

$$e^+e^- \to q\,\bar{q} \to h_1\,h_2\,X$$

$$h = \pi,\ K$$

$\phi_1+\phi_2$ method:

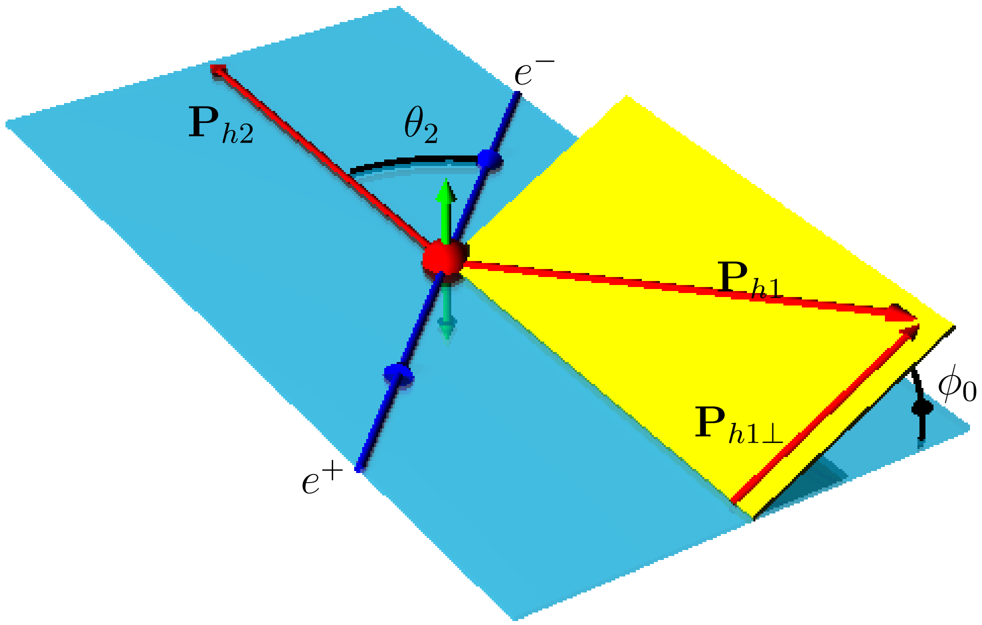
hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy



$$\mathcal{R}_{12} = \frac{N_{12}(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$

ϕ_0 method:

hadron 1 azimuthal angle with respect to hadron 2



$$\mathcal{R}_0 = \frac{N_0(\phi_0)}{\langle N_0 \rangle}$$

Double-ratios

But! Acceptance and radiation effects also contribute to azimuthal asymmetries!

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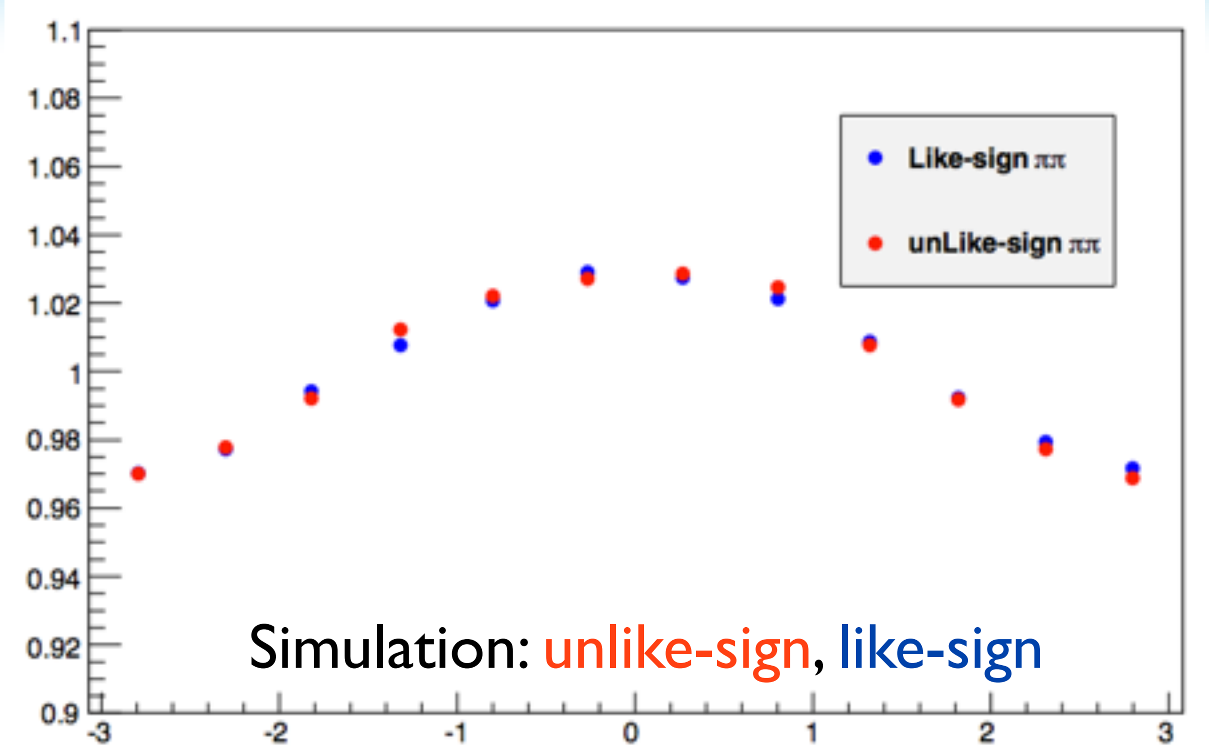
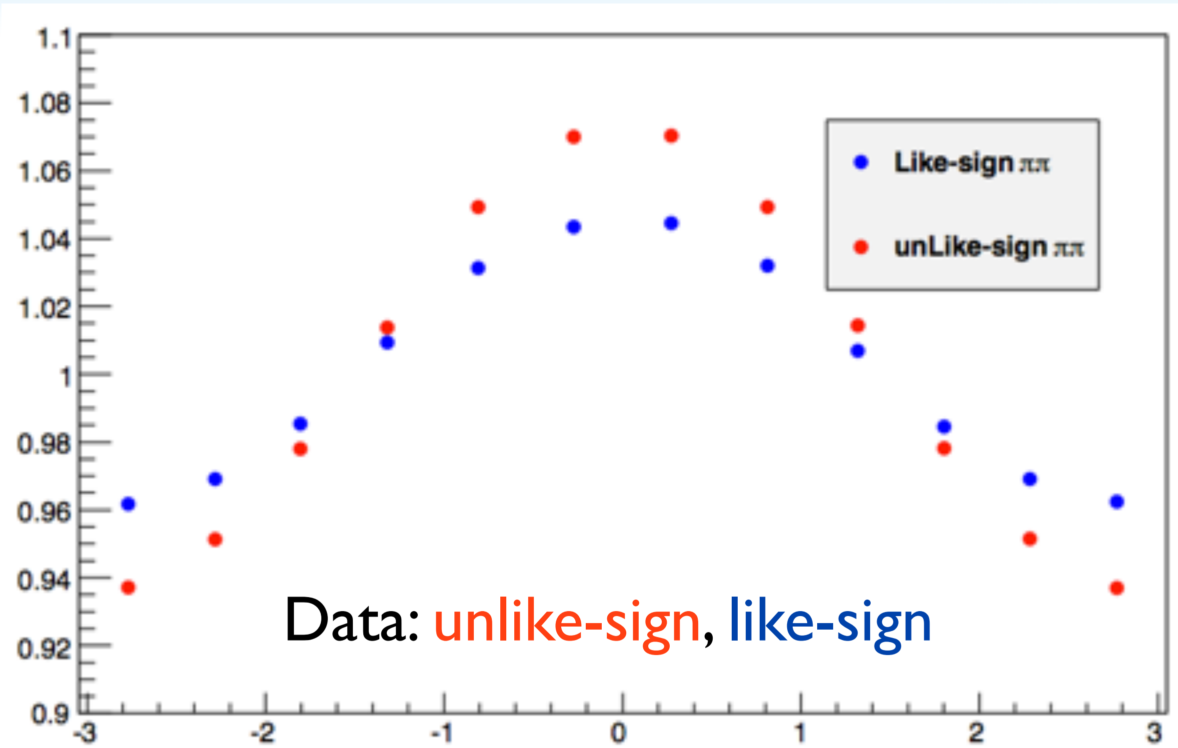
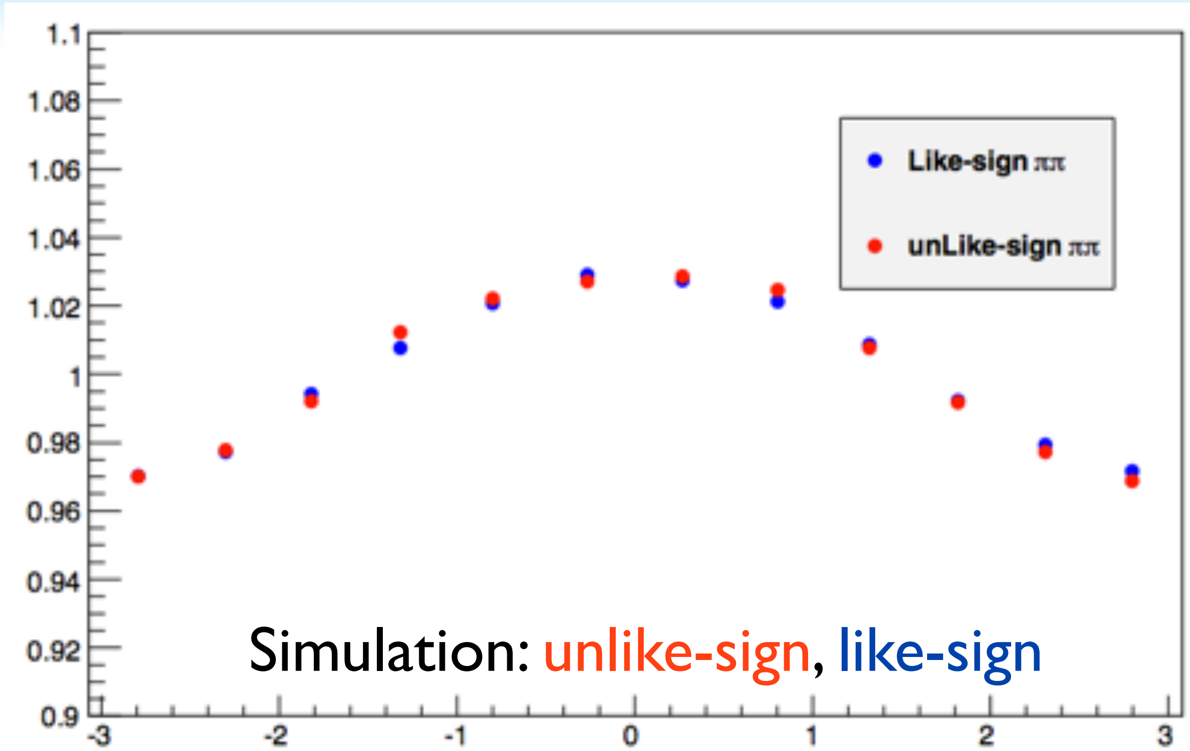
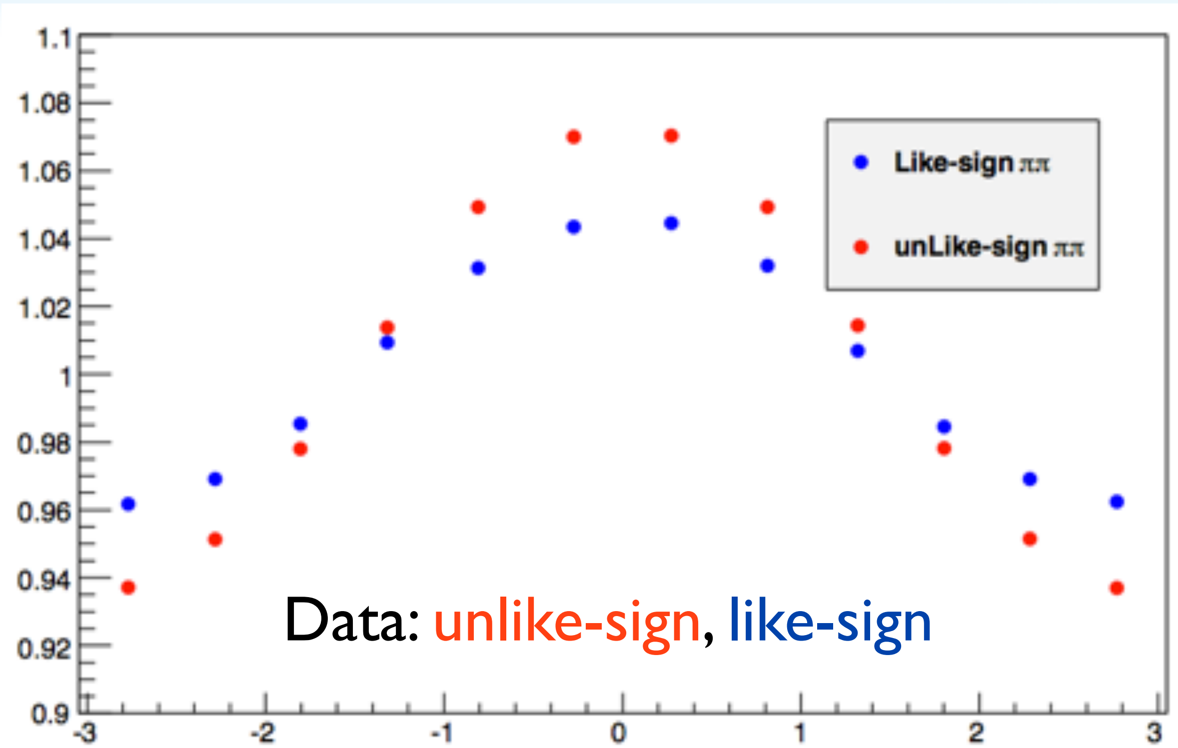


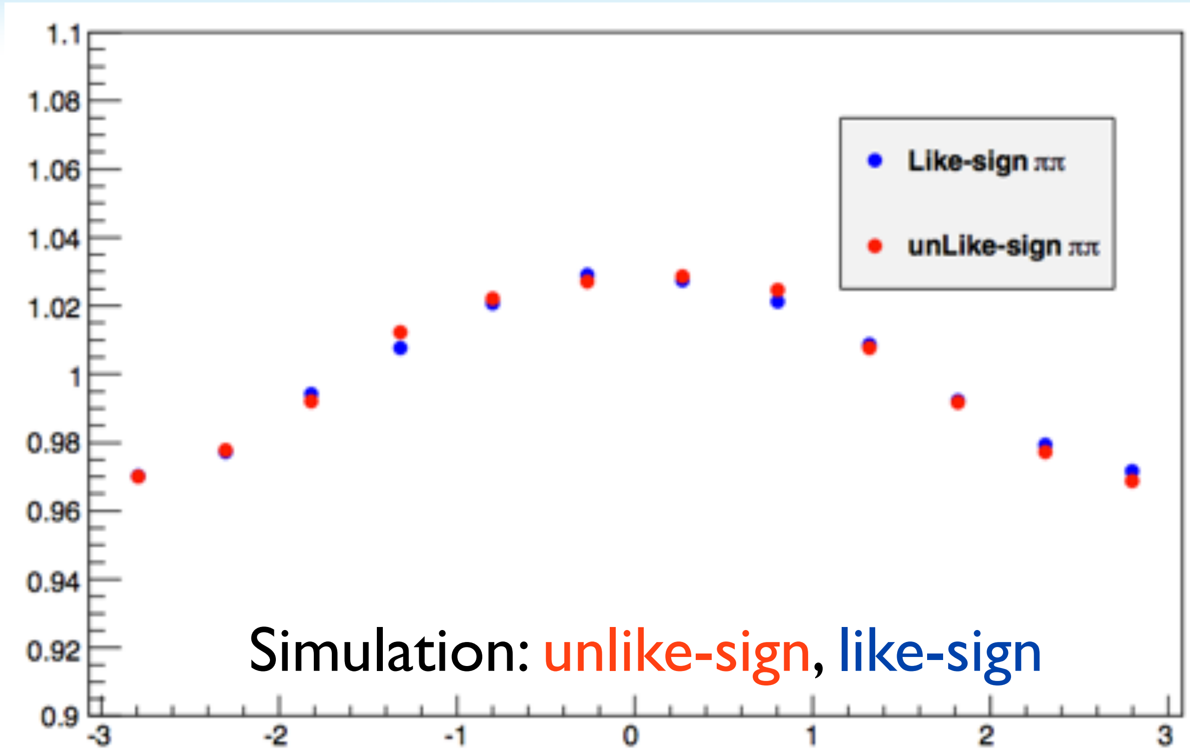
Double-ratios





Double-ratios





To reduce such non-Collins effects:
divide the sample of hadron couples in unlike-sign and like-sign (or All-charges), and extract the asymmetries of the super ratios between these 2 samples:

Unlike-sign couples / Like-sign couples

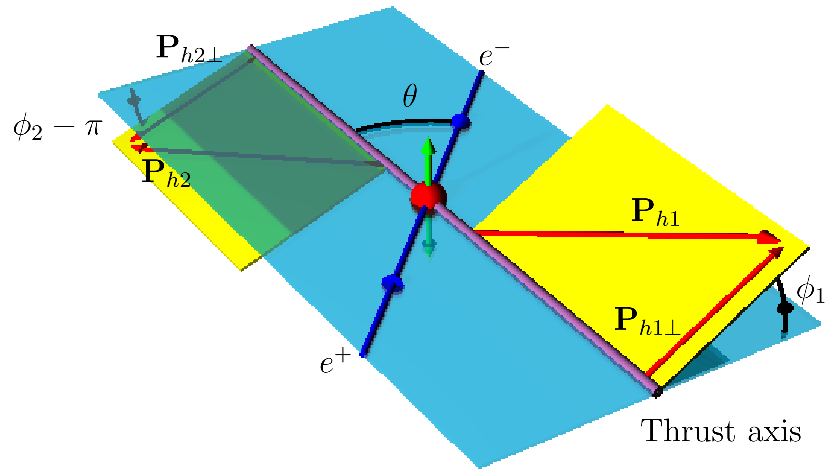
$$\mathcal{D}_{ul}^{h_1h_2} = \mathcal{R}^U/\mathcal{R}^L$$

Unlike-sign couples / All charges

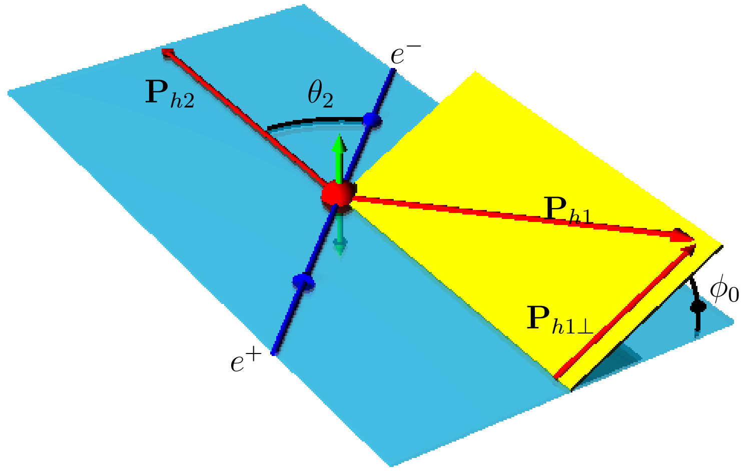
$$\mathcal{D}_{uc}^{h_1h_2} = \mathcal{R}^U/\mathcal{R}^C$$

Double-ratios

$\phi_1+\phi_2$ method



ϕ_0 method



$$\sigma \sim \mathcal{M}_{12}\left(1+\frac{\sin^2\theta_T}{1+\cos^2\theta_T}\,cos(\phi_1+\phi_2)\,\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\right)$$

$$\sigma \sim \mathcal{M}_0\left(1+\frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,cos(2\phi_0)\,\mathcal{F}\left[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\right]\right)$$

$$\mathcal{D}_{ul}^{h_1h_2}=\mathcal{R}^U/\mathcal{R}^L$$

Fitted by

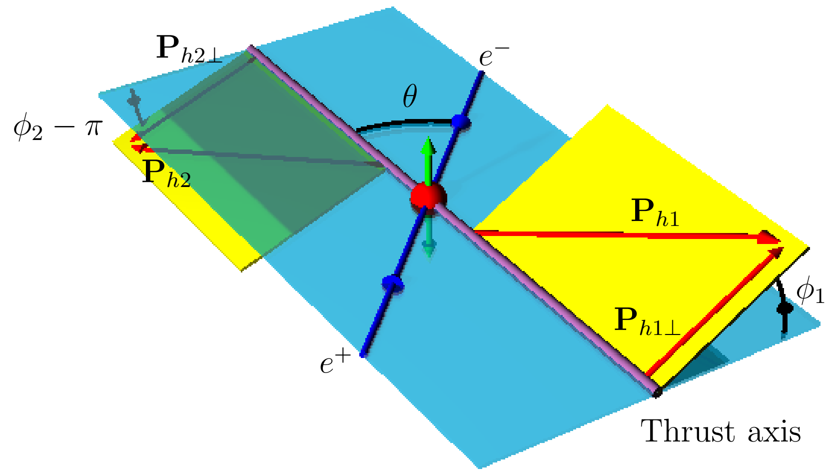
$$\mathcal{D}_{uc}^{h_1h_2}=\mathcal{R}^U/\mathcal{R}^C$$

$$\mathcal{B}_{12}(1+\mathcal{A}_{12}\cos(\phi_1+\phi_2))$$

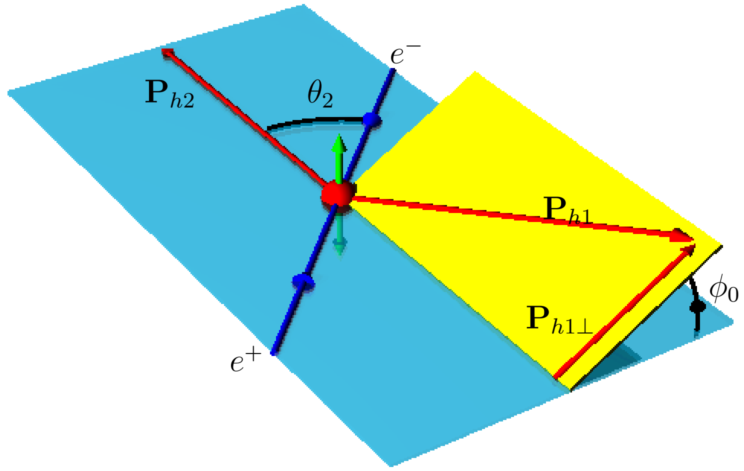
$$\mathcal{B}_0(1+\mathcal{A}_0\cos(2\phi_0))$$

Double-ratios

$\phi_1+\phi_2$ method



ϕ_0 method



$$\sigma \sim \mathcal{M}_{12}\Big(1+\frac{\sin^2\theta_T}{1+\cos^2\theta_T}\cos(\phi_1+\phi_2)\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

$$\sigma \sim \mathcal{M}_0\Big(1+\frac{\sin^2\theta_2}{1+\cos^2\theta_2}\cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$

$$A_{12}=\frac{\sin^2\theta}{1+\cos^2\theta}\,\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}$$

Fitted by

$$A_0=\frac{\sin^2\theta}{1+\cos^2\theta}\,\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]$$

$$\mathcal{B}_{12}(1+\mathcal{A}_{12}\cos(\phi_1+\phi_2))$$

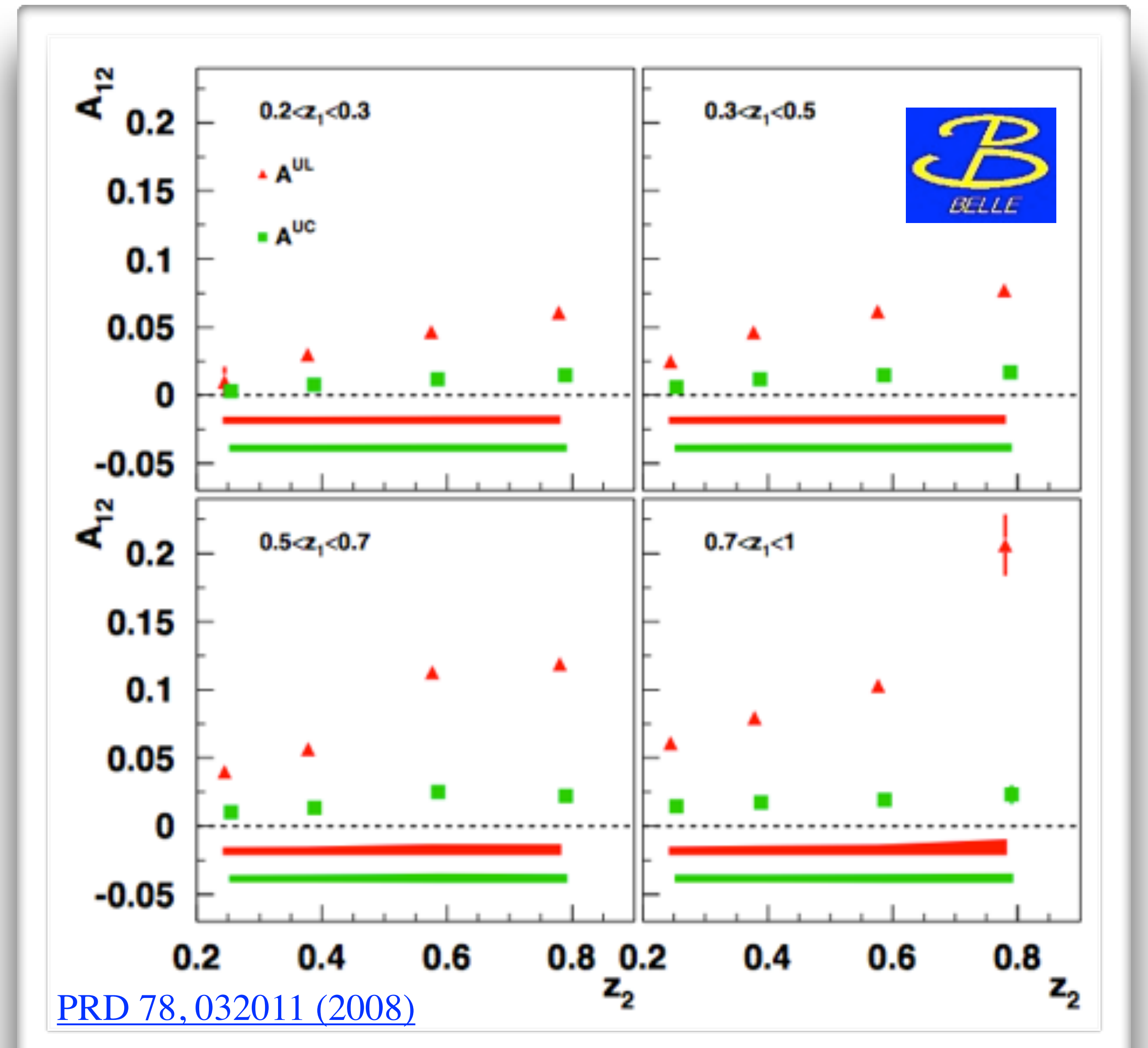
$$\mathcal{B}_0(1+\mathcal{A}_0\cos(2\phi_0))$$

Belle publications
PRL 96,232002, (2006)
PRD 78, 032011 (2008)

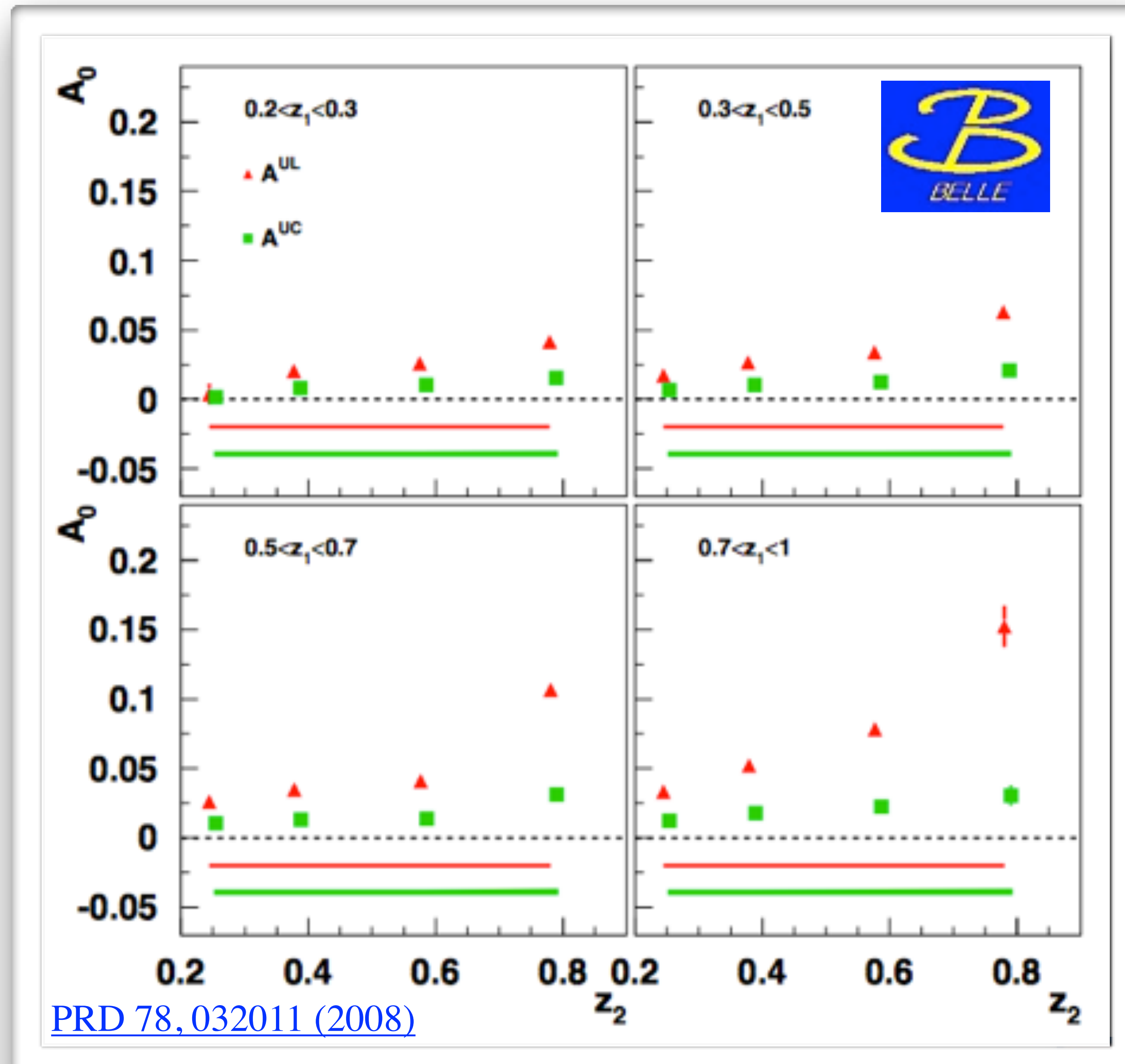
Published results: $\pi\pi$

$\phi_1+\phi_2$ method

ϕ_0 method

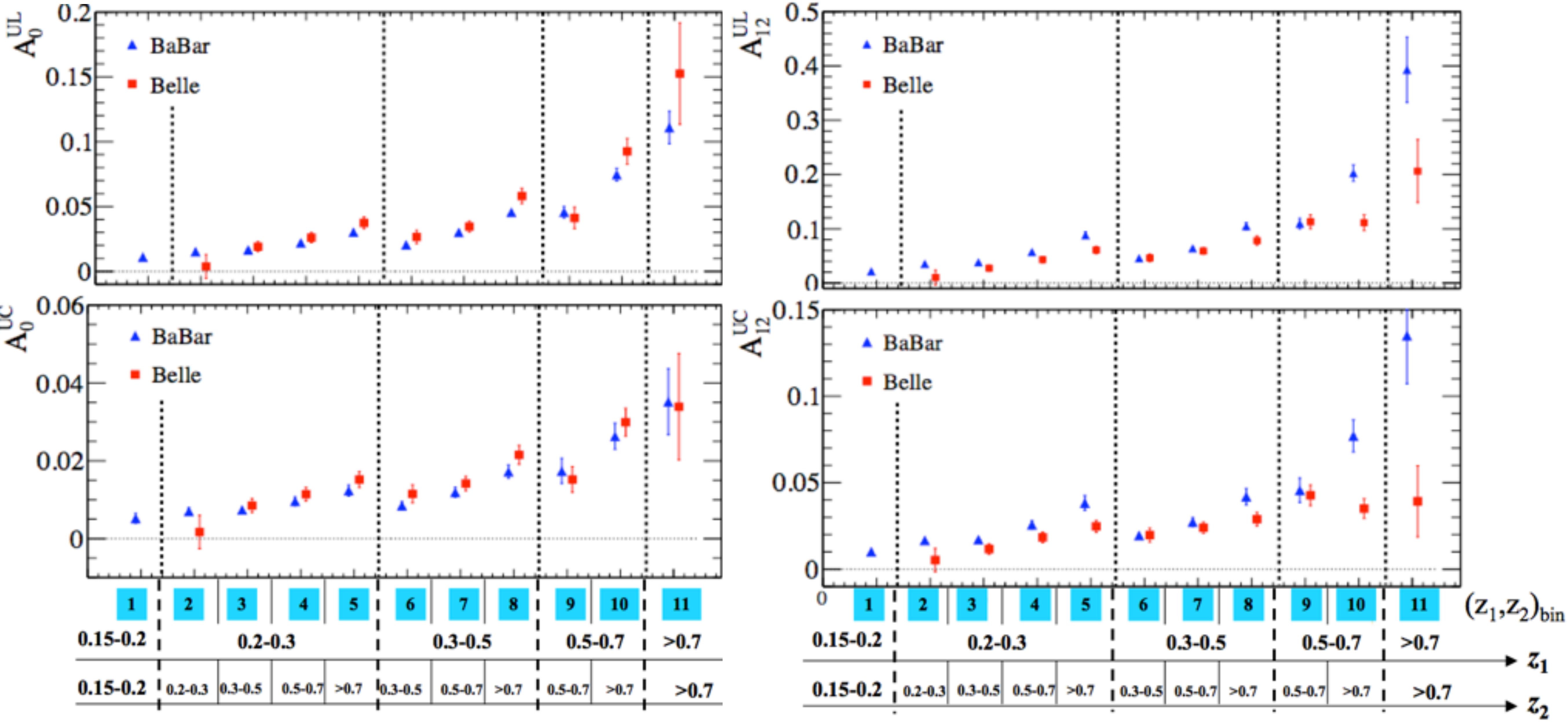


PRD 78, 032011 (2008)

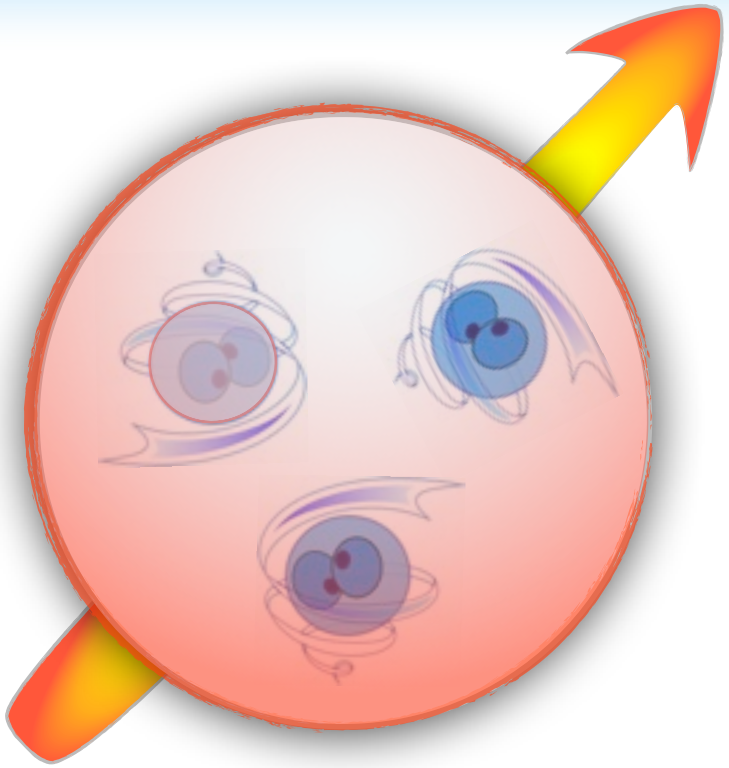


PRD 78, 032011 (2008)

Belle vs. Babar

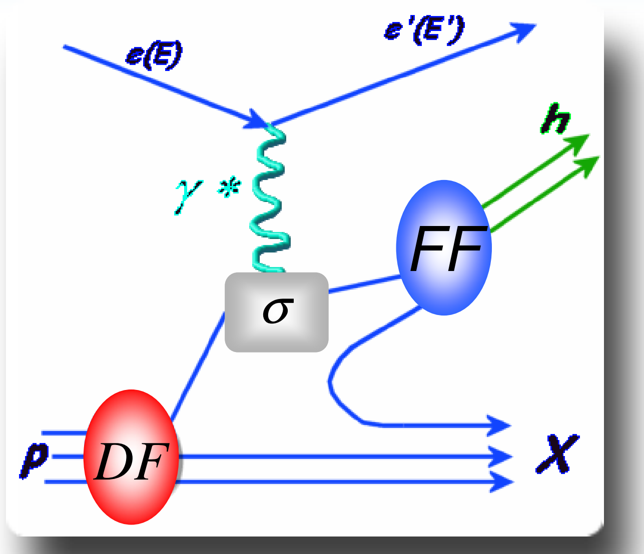


SIDIS



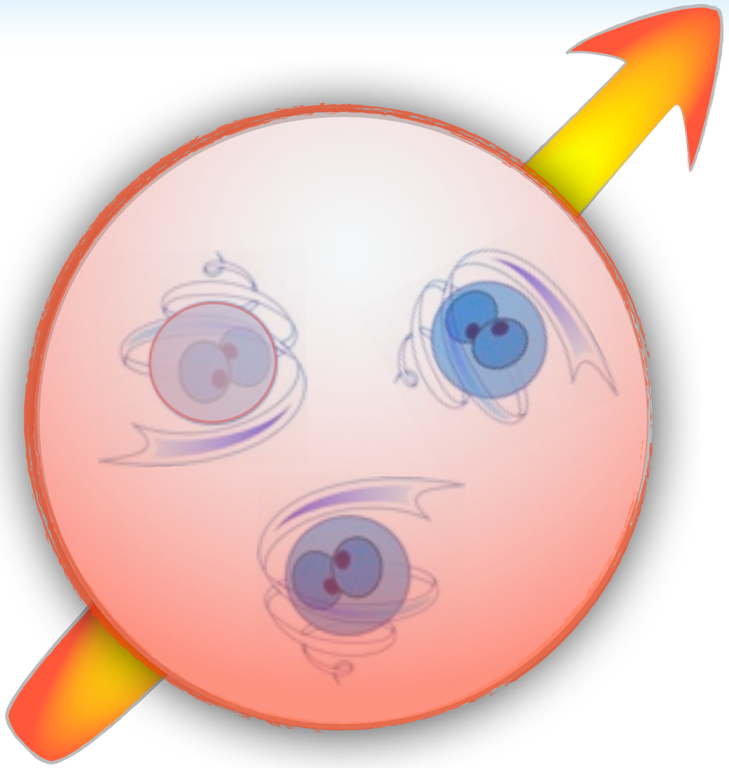
Chiral odd!

$$\underbrace{\overset{\text{chiral odd}}{X} \otimes \overset{\text{chiral odd}}{H_1^\perp}}_{\text{chiral even}}$$



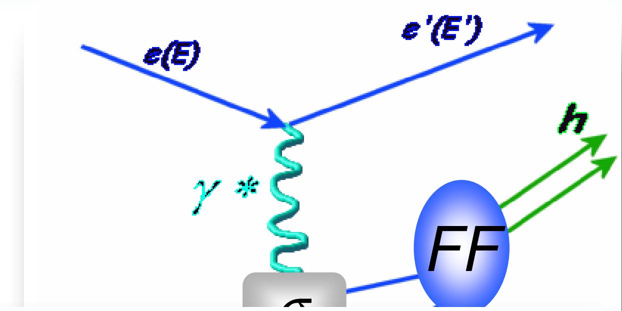
		quark		
		U	L	T
nucleon	U	f_1		$h_1^\perp$
	L		g_1	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

SIDIS



Chiral odd!

$$\underbrace{\underset{\text{chiral odd}}{X} \otimes \underset{\text{chiral odd}}{H_1^{\perp}}}_{\text{chiral even}}$$



$$\frac{d\sigma^h}{dx\, dy\, d\phi_S\, dz\, d\phi\, d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

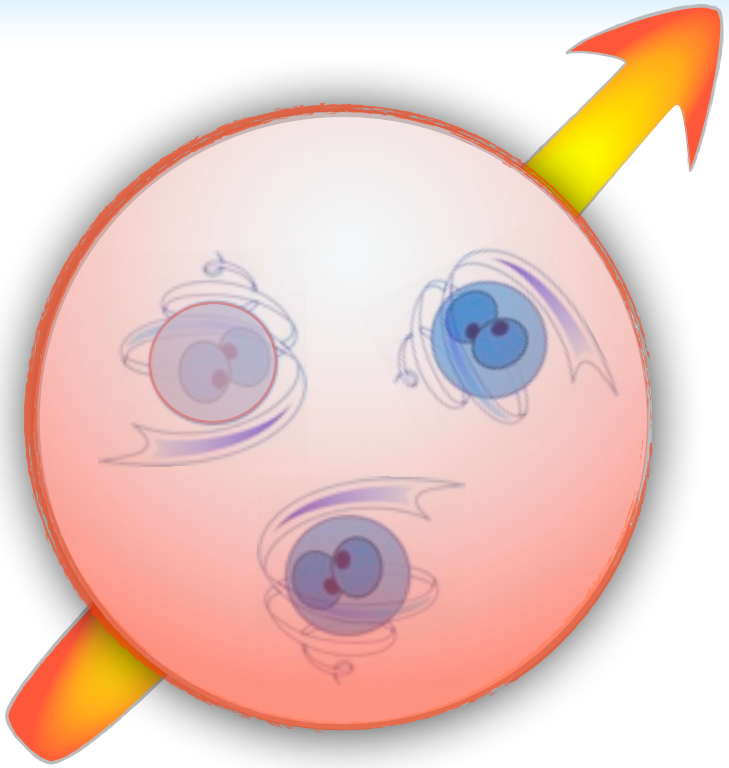
$h_1^{\perp} \otimes H_1^{\perp}$

$$\begin{aligned}
\Big\{ \quad & \Big[F_{UU,T} + \epsilon F_{UU,L} \\
& + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi) F_{UU}^{\cos(\phi)} + \boxed{\epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)}}\Big] \\
+ \quad \lambda_l & \Big[\sqrt{2\epsilon(1-\epsilon)}\sin(\phi) F_{LU}^{\sin(\phi)}\Big] \\
+ S_L \quad & \Big[\sqrt{2\epsilon(1+\epsilon)}\sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)}\Big] \\
+ S_L\, \lambda_l & \Big[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi) F_{LL}^{\cos(\phi)}\Big] \\
+ S_T \quad & \Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right) \\
& \boxed{+\epsilon \sin(\phi+\phi_S) F_{UT}^{\sin(\phi+\phi_S)}} + \epsilon \sin(3\phi-\phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\
& + \sqrt{2\epsilon(1+\epsilon)}\sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\
& + \sqrt{2\epsilon(1+\epsilon)}\sin(2\phi-\phi_S) F_{UT}^{\sin(2\phi-\phi_S)}\Big] \\
+ S_T\, \lambda_l & \Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\
& + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\
& + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S) F_{LT}^{\cos(2\phi-\phi_S)}\Big] \Big\}
\end{aligned}$$

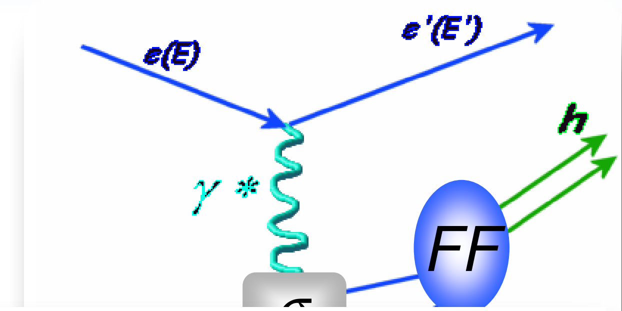
$h_1 \otimes H_1^{\perp}$

		quark		
		U	L	T
nucleon	U	f_1		$h_1^{\perp}$
	L		g_1	$h_{1L}^{\perp}$
	T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	h_1 $h_{1T}^{\perp}$

SIDIS



u-dominance



$h_1^\perp \otimes H_1^\perp$

$h_1 \otimes H_1^\perp$

$$\frac{d\sigma^h}{dx\,dy\,d\phi_S\,dz\,d\phi\,d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

$$\Big\{ \Big[F_{\mathrm{UU,T}} + \epsilon F_{\mathrm{UU,L}} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi)F_{\mathrm{UU}}^{\cos(\phi)} + \boxed{\epsilon\cos(2\phi)F_{\mathrm{UU}}^{\cos(2\phi)}}\Big]$$

$$+ \lambda_l \Big[\sqrt{2\epsilon(1-\epsilon)}\sin(\phi)F_{\mathrm{LU}}^{\sin(\phi)}\Big]$$

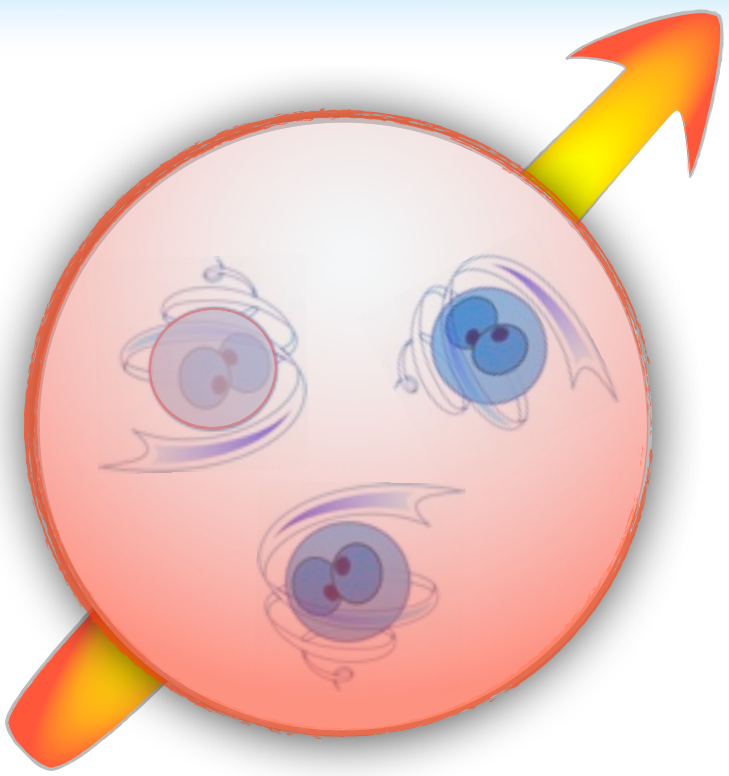
$$+ S_L \Big[\sqrt{2\epsilon(1+\epsilon)}\sin(\phi)F_{\mathrm{UL}}^{\sin(\phi)} + \epsilon\sin(2\phi)F_{\mathrm{UL}}^{\sin(2\phi)}\Big]$$

$$+ S_L\,\lambda_l \Big[\sqrt{1-\epsilon^2}F_{\mathrm{LL}} + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi)F_{\mathrm{LL}}^{\cos(\phi)}\Big]$$

$$+ S_T \Big[\sin(\phi-\phi_S)\Big(F_{\mathrm{UT,T}}^{\sin(\phi-\phi_S)} + \epsilon F_{\mathrm{UT,L}}^{\sin(\phi-\phi_S)}\Big) + \boxed{\epsilon\sin(\phi+\phi_S)F_{\mathrm{UT}}^{\sin(\phi+\phi_S)}} + \epsilon\sin(3\phi-\phi_S)F_{\mathrm{UT}}^{\sin(3\phi-\phi_S)} + \sqrt{2\epsilon(1+\epsilon)}\sin(\phi_S)F_{\mathrm{UT}}^{\sin(\phi_S)} + \sqrt{2\epsilon(1+\epsilon)}\sin(2\phi-\phi_S)F_{\mathrm{UT}}^{\sin(2\phi-\phi_S)}\Big]$$

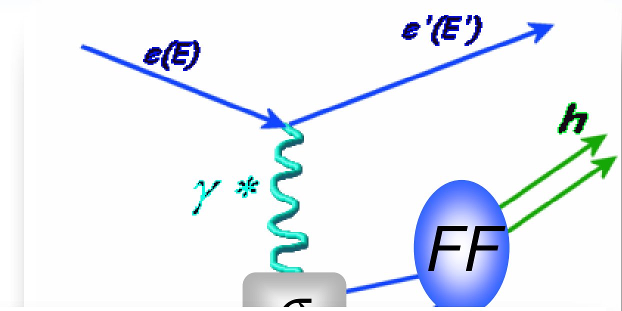
$$+ S_T\,\lambda_l \Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{\mathrm{LT}}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi_S)F_{\mathrm{LT}}^{\cos(\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{\mathrm{LT}}^{\cos(2\phi-\phi_S)}\Big]\Big\}$$

SIDIS



u-dominance

proton content: u u d



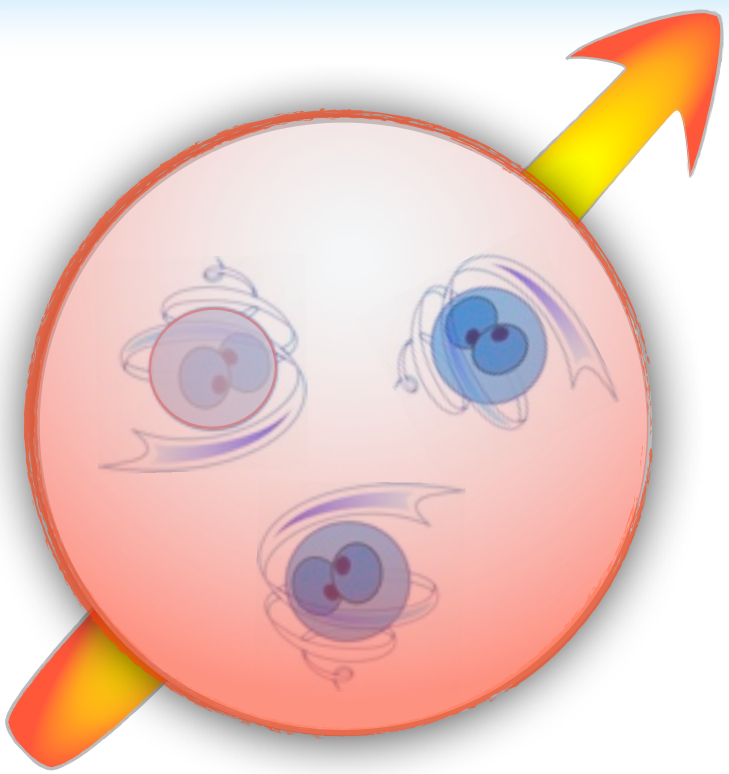
$$\frac{d\sigma^h}{dx\,dy\,d\phi_S\,dz\,d\phi\,d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

$$h_1^\perp \otimes H_1^\perp$$

$$\begin{aligned}
\Big\{ \quad & \Big[F_{UU,T} + \epsilon F_{UU,L} \\
& + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi) F_{UU}^{\cos(\phi)} + \boxed{\epsilon\cos(2\phi) F_{UU}^{\cos(2\phi)}}\Big] \\
+ \quad \lambda_l & \Big[\sqrt{2\epsilon(1-\epsilon)}\sin(\phi) F_{LU}^{\sin(\phi)}\Big] \\
+ S_L \quad & \Big[\sqrt{2\epsilon(1+\epsilon)}\sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon\sin(2\phi) F_{UL}^{\sin(2\phi)}\Big] \\
+ S_L\,\lambda_l & \Big[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi) F_{LL}^{\cos(\phi)}\Big] \\
+ S_T \quad & \Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right) \\
& \boxed{+\epsilon\sin(\phi+\phi_S) F_{UT}^{\sin(\phi+\phi_S)}} + \epsilon\sin(3\phi-\phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\
& + \sqrt{2\epsilon(1+\epsilon)}\sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\
& + \sqrt{2\epsilon(1+\epsilon)}\sin(2\phi-\phi_S) F_{UT}^{\sin(2\phi-\phi_S)}\Big] \\
+ S_T\,\lambda_l & \Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\
& + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\
& + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S) F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}
\end{aligned}$$

$$h_1 \otimes H_1^\perp$$

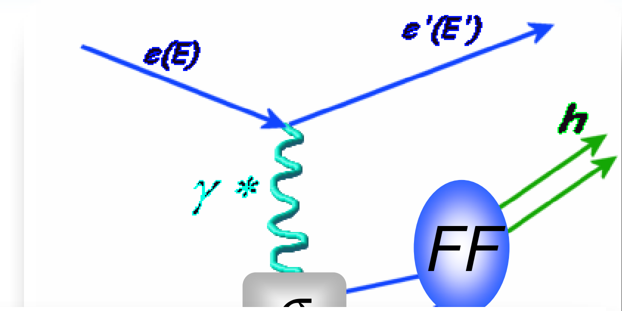
SIDIS



u-dominance

proton content: u u d

$$\sigma \propto e_q^2 ....\qquad e_d^2 = (1/3)^2 \qquad e_u^2 = (2/3)^2$$



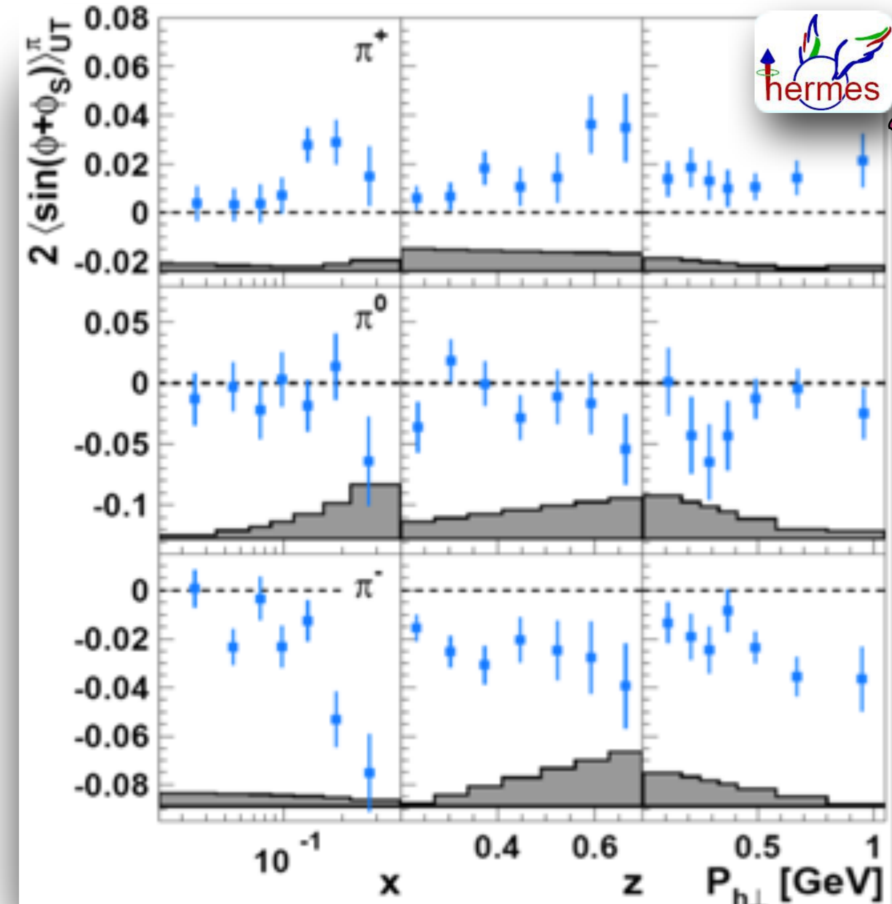
$$\frac{d\sigma^h}{dx\,dy\,d\phi_S\,dz\,d\phi\,d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

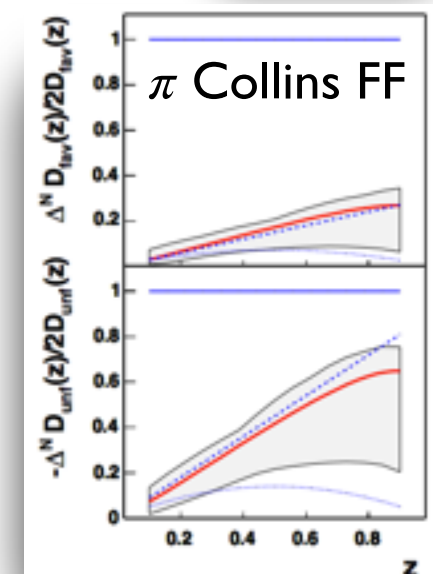
$$\begin{aligned}
\Big\{ & \Big[F_{UU,T} + \epsilon F_{UU,L} \\
& + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi) F_{UU}^{\cos(\phi)} + \boxed{\epsilon\cos(2\phi) F_{UU}^{\cos(2\phi)}}\Big] \qquad h_1^\perp \otimes H_1^\perp \\
+ & \lambda_l \Big[\sqrt{2\epsilon(1-\epsilon)}\sin(\phi) F_{LU}^{\sin(\phi)}\Big] \\
+ & S_L \Big[\sqrt{2\epsilon(1+\epsilon)}\sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon\sin(2\phi) F_{UL}^{\sin(2\phi)}\Big] \\
+ & S_L\,\lambda_l \Big[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi) F_{LL}^{\cos(\phi)}\Big] \\
+ & S_T \Big[\sin(\phi-\phi_S)\Big(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\Big) \\
& + \boxed{\epsilon\sin(\phi+\phi_S) F_{UT}^{\sin(\phi+\phi_S)}} + \epsilon\sin(3\phi-\phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \qquad h_1 \otimes H_1^\perp \\
& + \sqrt{2\epsilon(1+\epsilon)}\sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\
& + \sqrt{2\epsilon(1+\epsilon)}\sin(2\phi-\phi_S) F_{UT}^{\sin(2\phi-\phi_S)}\Big] \\
+ & S_T\,\lambda_l \Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\
& + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\
& + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S) F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}
\end{aligned}$$

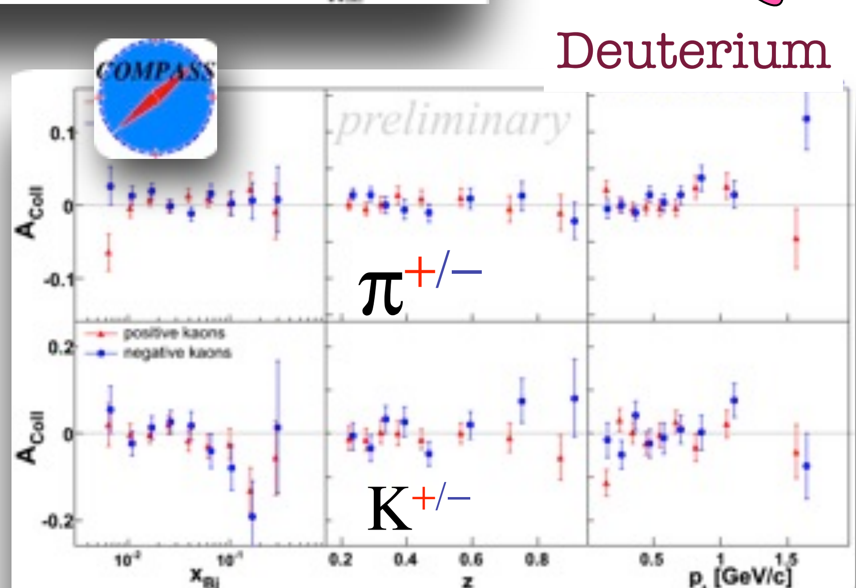
Collins amplitudes in SIDIS

$$A_{UT} \propto h_1 \otimes H_1^{\perp}$$

[Airapetian *et al.*, Phys. Lett. B 693 (2010) 11-16]







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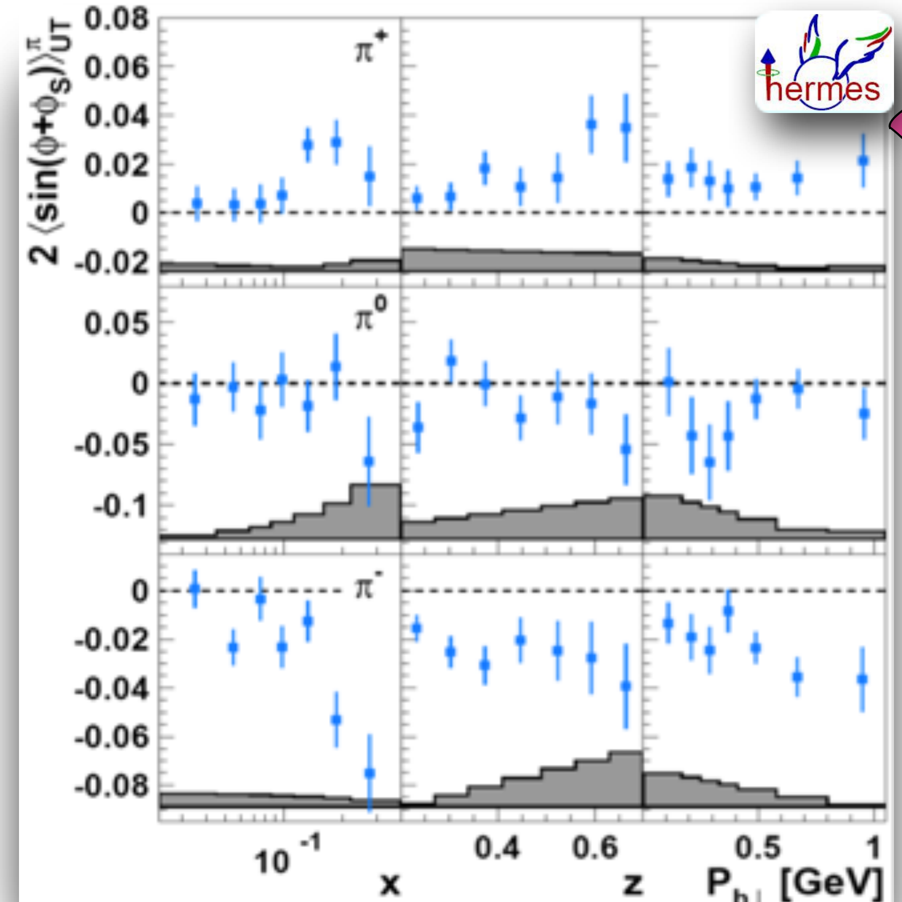
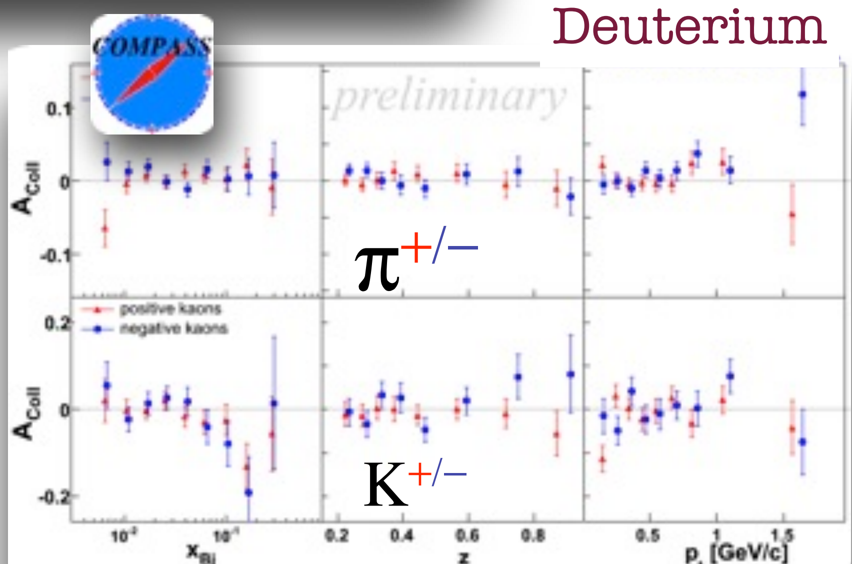
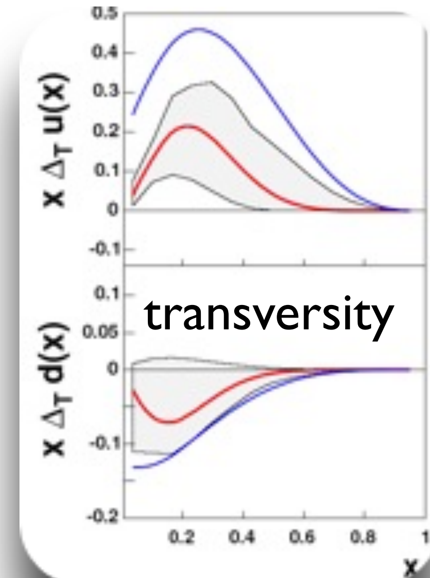
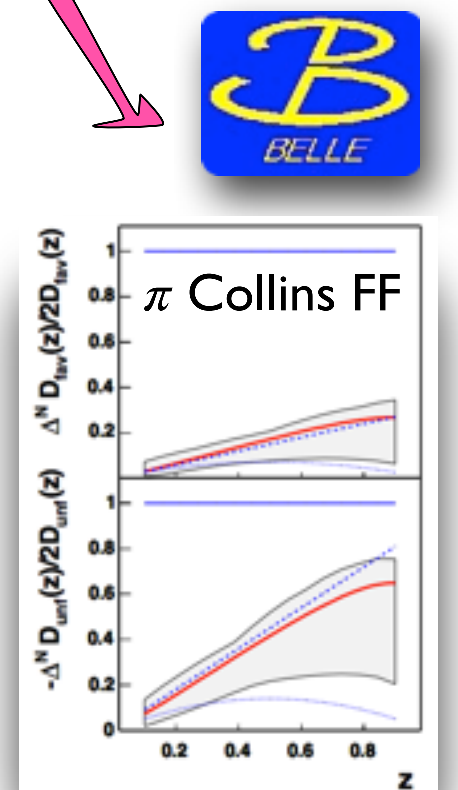


Deuterium



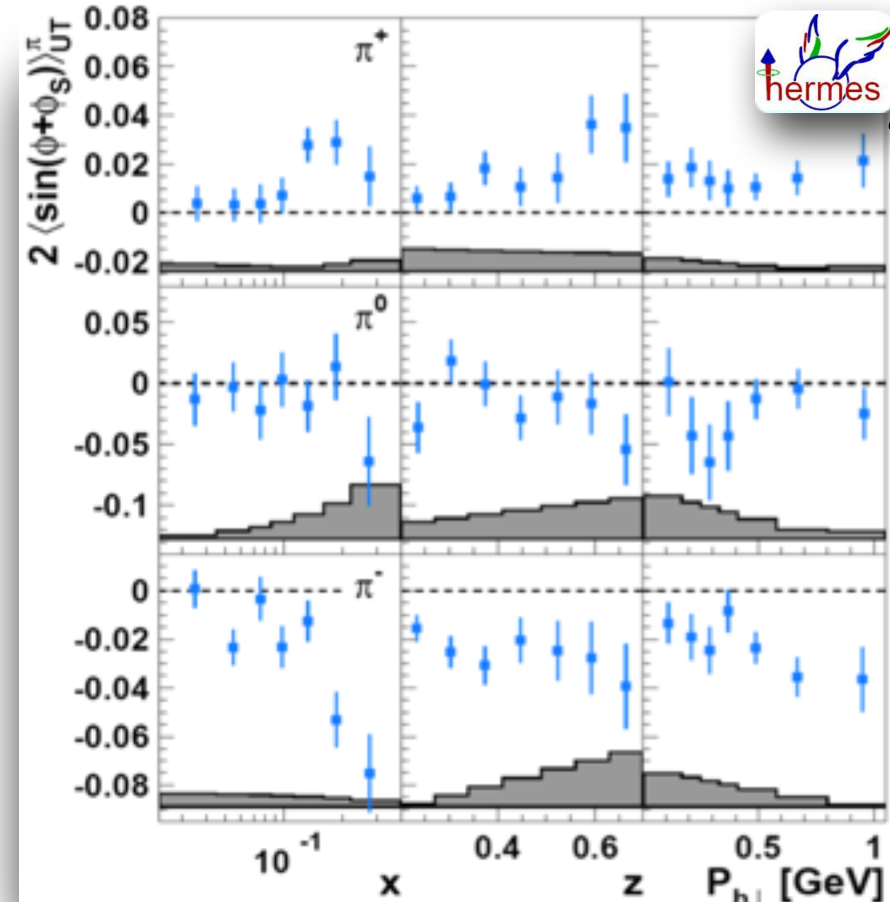
Anselmino et al.
Phys.Rev. D75 (2007)





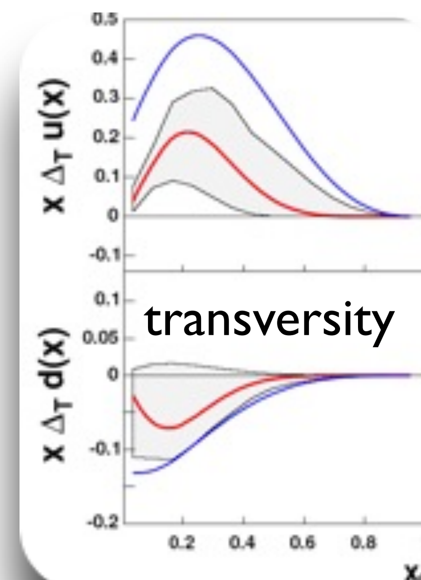
Collins amplitudes in SIDIS

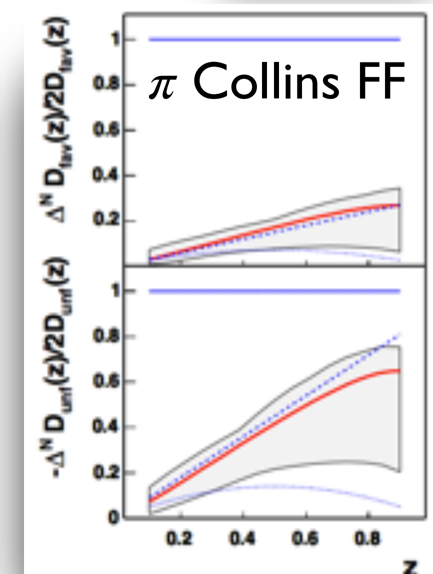
[Airapetian *et al.*, Phys. Lett. B 693 (2010) 11-16]



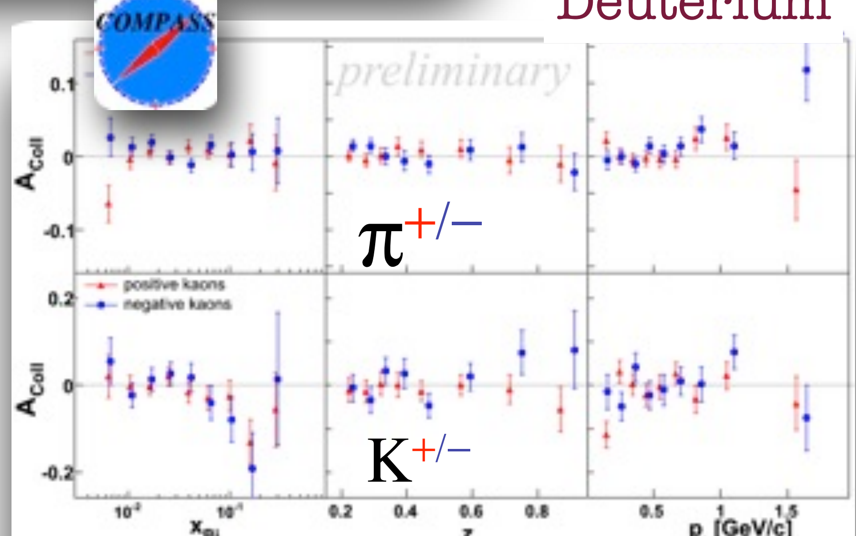
$$A_{UT} \propto h_1 \otimes H_1^{\perp}$$

Anselmino et al.
Phys.Rev. D75 (2007)

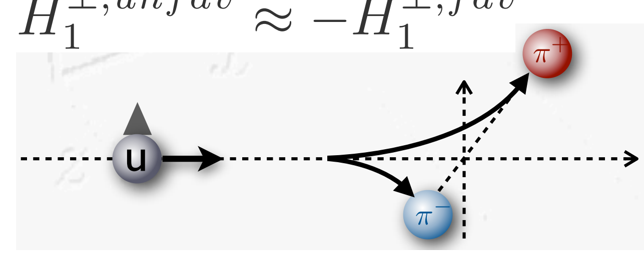




Deuterium

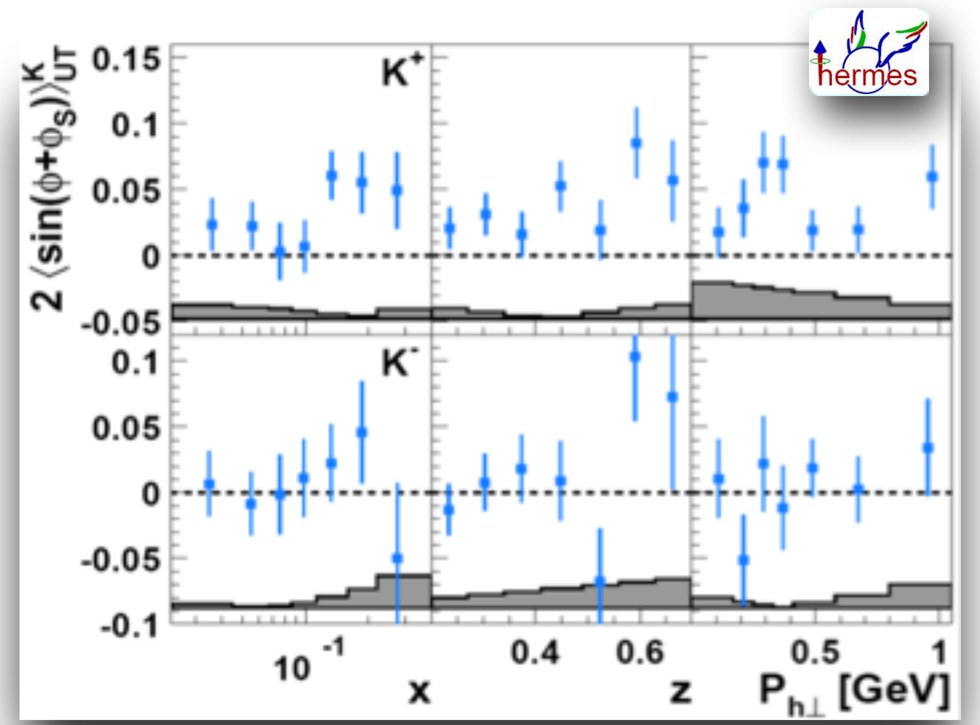


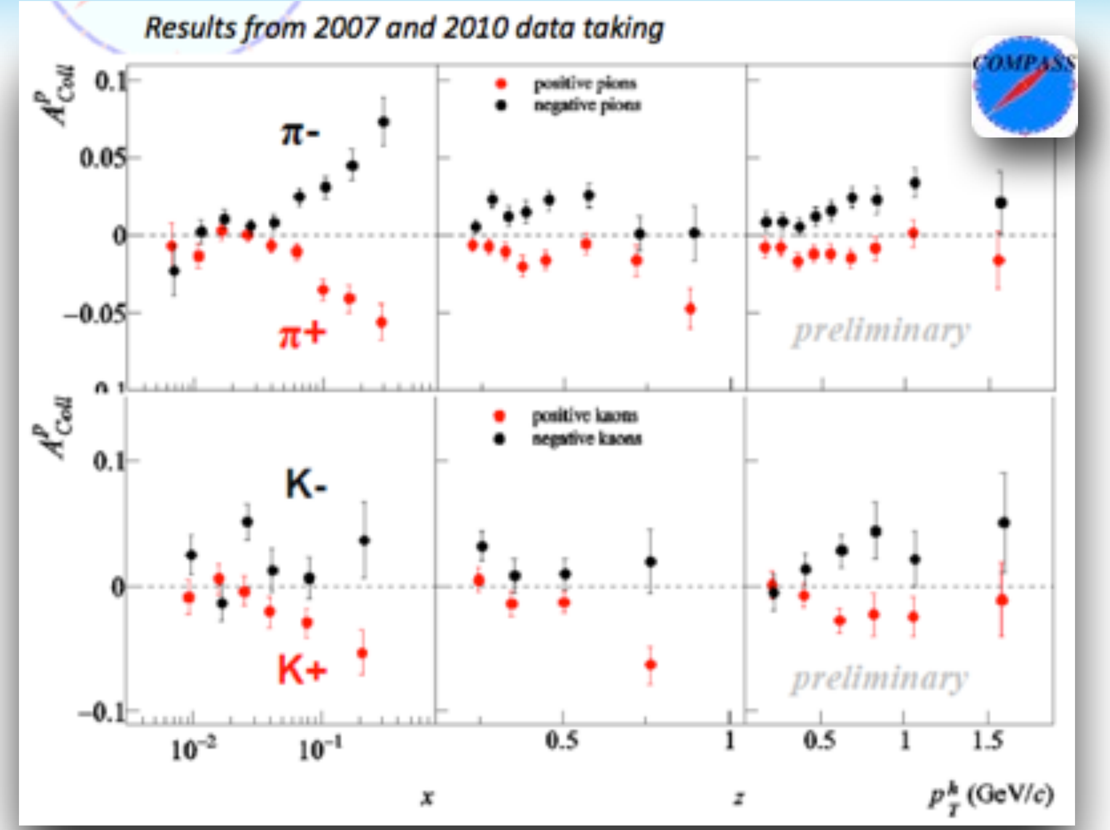
$$H_1^{\perp,unfav} \approx -H_1^{\perp,fav}$$



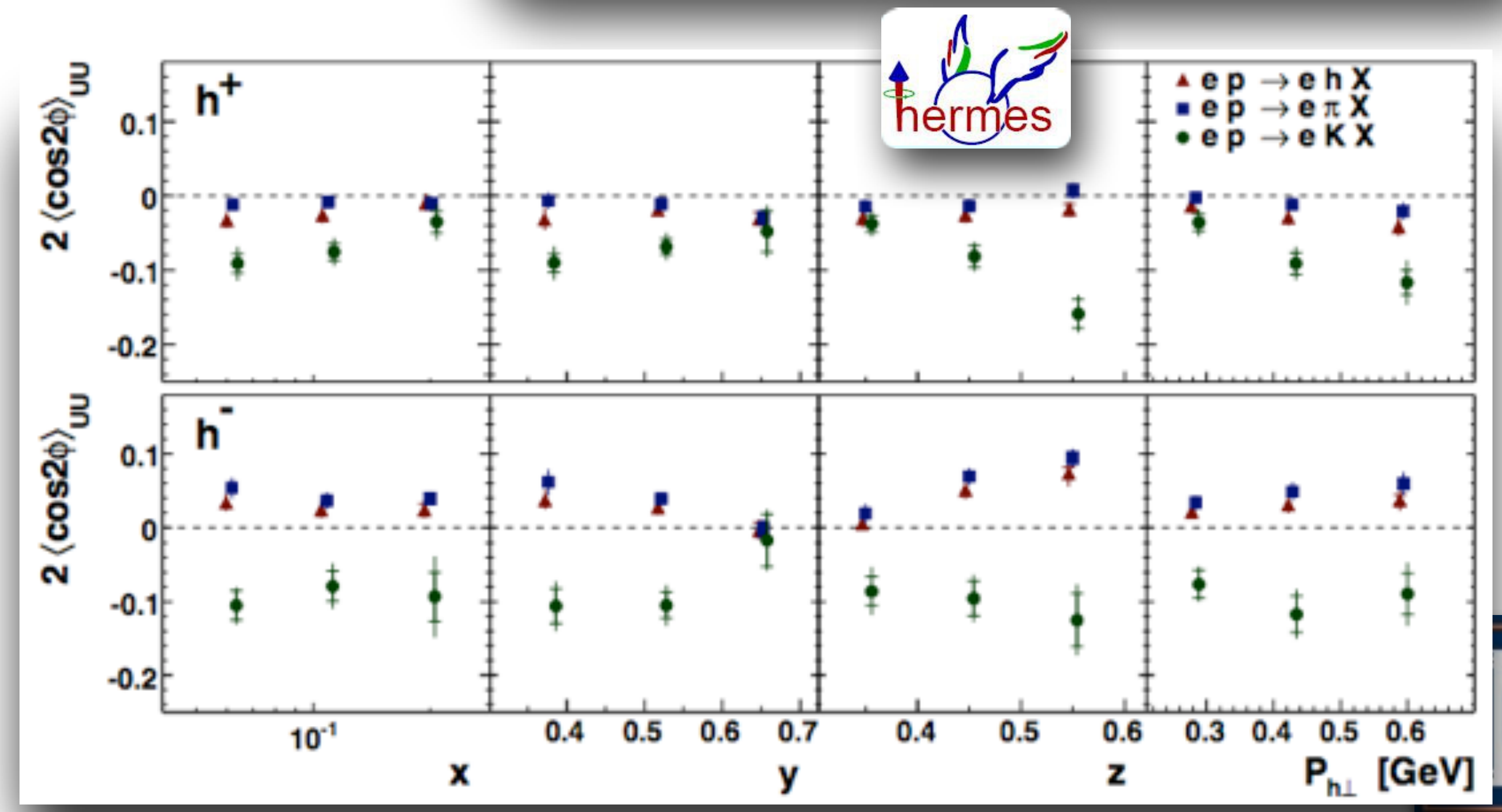
Collins amplitudes in SIDIS

$$A_{UT} \propto h_1 \otimes H_1^{\perp}$$



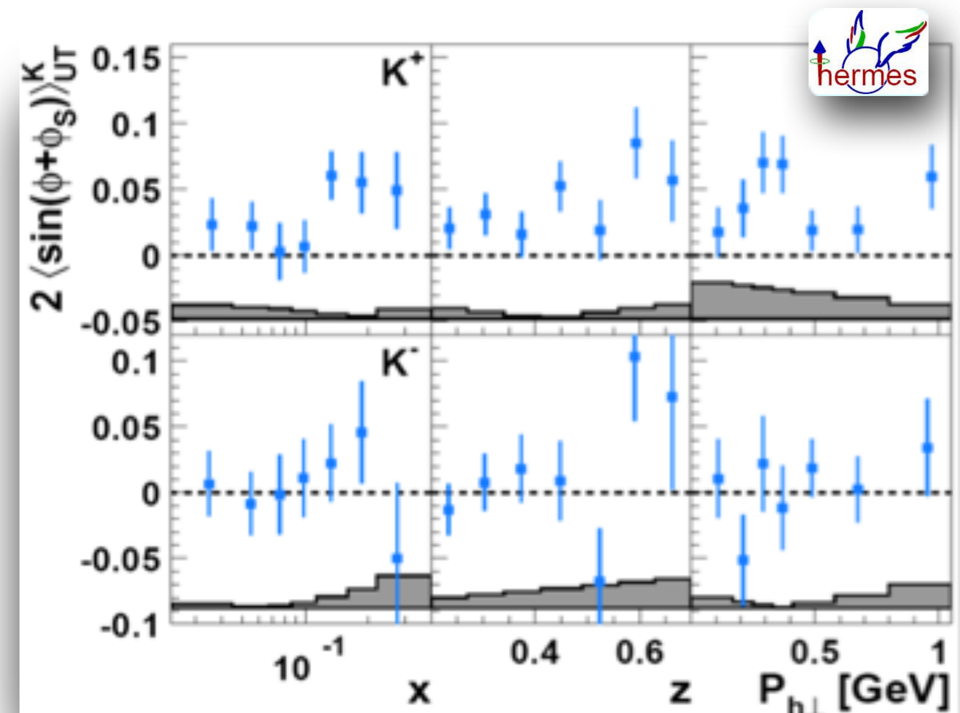


$$A_{UU} \propto h_1^{\perp} \otimes H_1^{\perp}$$

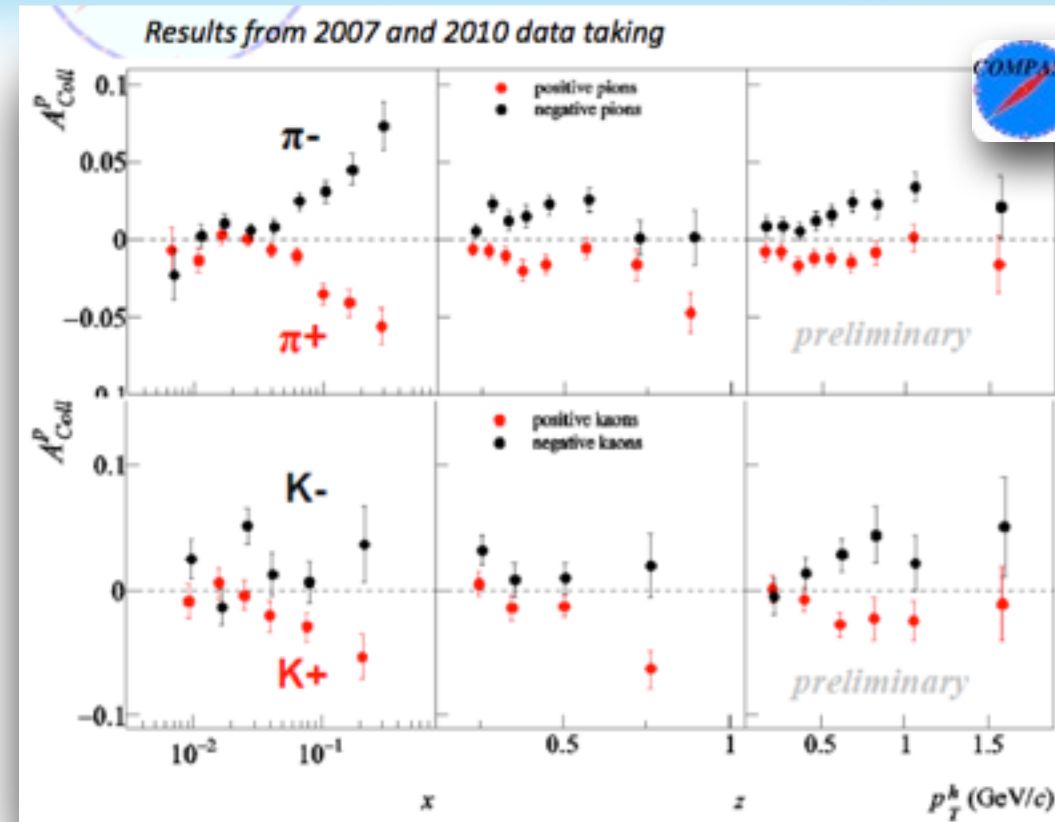


Collins amplitudes in SIDIS

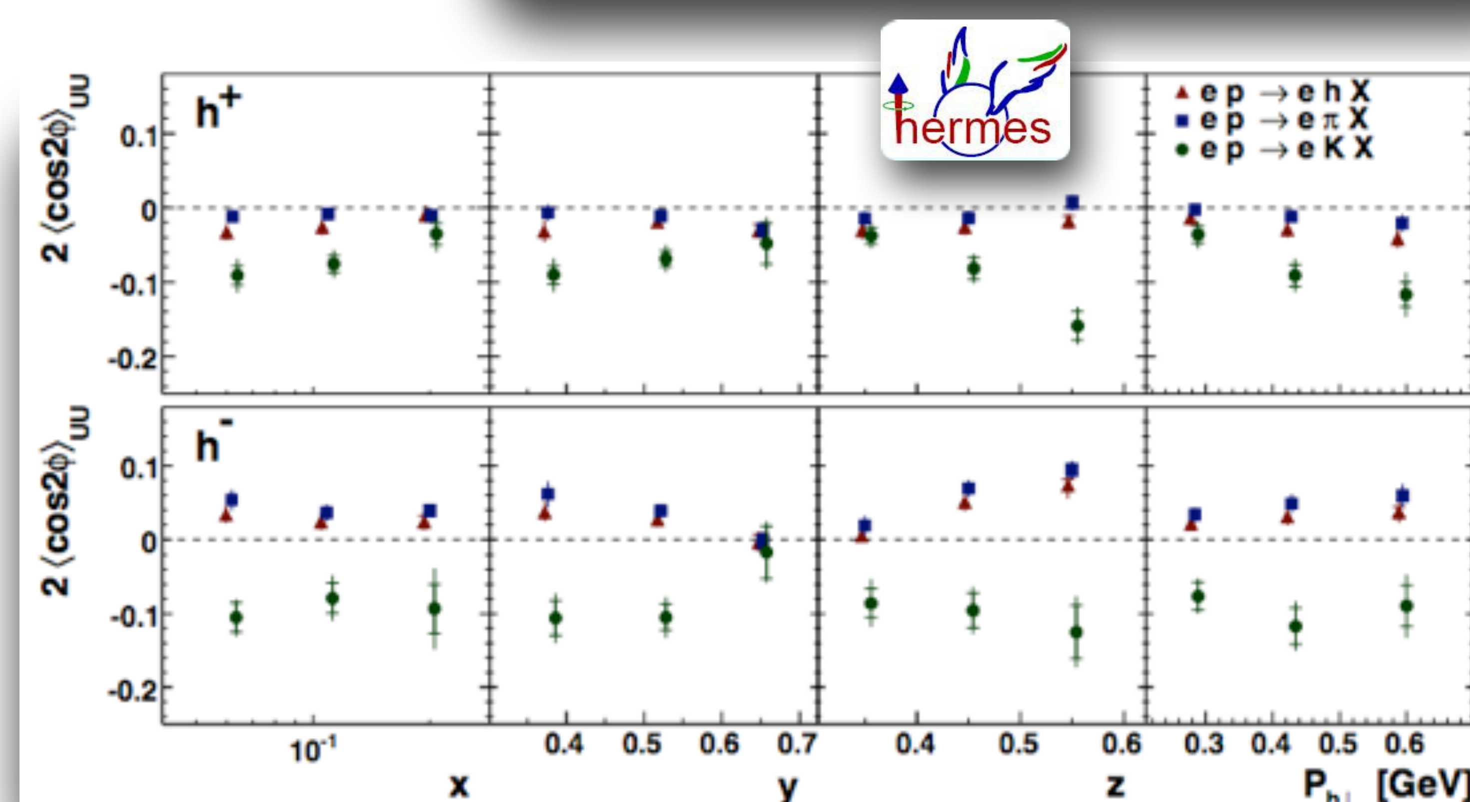
$$A_{UT} \propto h_1 \otimes H_1^\perp$$



K⁺ amplitudes larger than π^+?



$$A_{UU} \propto h_1^\perp \otimes H_1^\perp$$



More recently...

	z	q_T	$\sin^2\theta/(1+\cos^2\theta)$	p_T
$\mathcal{D}^{\pi\pi}_{ul} = \mathcal{R}^{U\pi\pi}/\mathcal{R}^{L\pi\pi}$	√	√	√	New!
$\mathcal{D}^{\pi k}_{ul} = \mathcal{R}^{U\pi k}/\mathcal{R}^{L\pi k}$	New!	New!	New!	New!
$\mathcal{D}^{kk}_{ul} = \mathcal{R}^{Ukk}/\mathcal{R}^{Lkk}$	New!	New!	New!	New!

	z	q_T	$\sin^2\theta/(1+\cos^2\theta)$	p_T
$\mathcal{D}^{\pi\pi}_{uc} = \mathcal{R}^{U\pi\pi}/\mathcal{R}^{C\pi\pi}$	√	√	√	New!
$\mathcal{D}^{\pi k}_{uc} = \mathcal{R}^{U\pi k}/\mathcal{R}^{C\pi k}$	New!	New!	New!	New!
$\mathcal{D}^{kk}_{uc} = \mathcal{R}^{Ukk}/\mathcal{R}^{Ckk}$	New!	New!	New!	New!

More recently...

	z	q_T	$\sin^2\theta/(1+\cos^2\theta)$	p_T
$\mathcal{D}^{\pi\pi}_{ul} = \mathcal{R}^{U\pi\pi}/\mathcal{R}^{L\pi\pi}$	√	√	√	New!
$\mathcal{D}^{\pi k}_{ul} = \mathcal{R}^{U\pi k}/\mathcal{R}^{L\pi k}$	New!	New!	New!	New!
$\mathcal{D}^{kk}_{ul} = \mathcal{R}^{Ukk}/\mathcal{R}^{Lkk}$	New!	New!	New!	New!

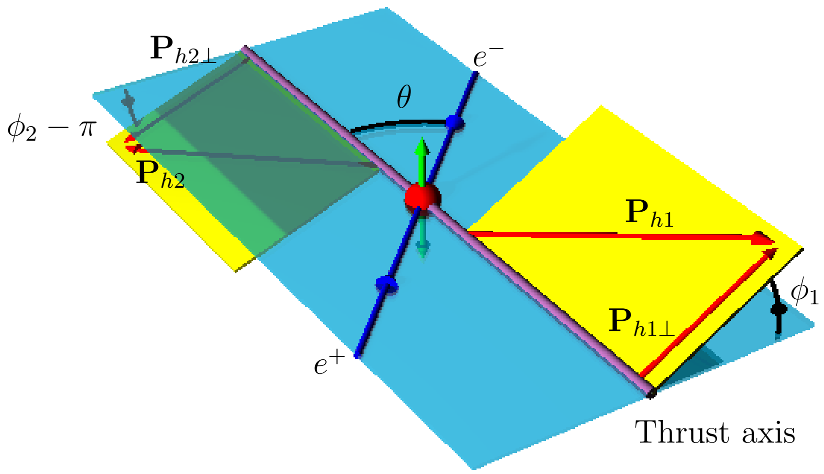
	z	q_T	$\sin^2\theta/(1+\cos^2\theta)$	p_T
$\mathcal{D}^{\pi\pi}_{uc} = \mathcal{R}^{U\pi\pi}/\mathcal{R}^{C\pi\pi}$	√	√	√	New!
$\mathcal{D}^{\pi k}_{uc} = \mathcal{R}^{U\pi k}/\mathcal{R}^{C\pi k}$	New!	New!	New!	New!
$\mathcal{D}^{kk}_{uc} = \mathcal{R}^{Ukk}/\mathcal{R}^{Ckk}$	New!	New!	New!	New!

Word of caution: this analysis is mainly aimed at kaons, so kinematic cuts and binning are optimized for kaons, and the same values used for pion too.

→ $\pi\pi$ results cannot be compared directly to published results

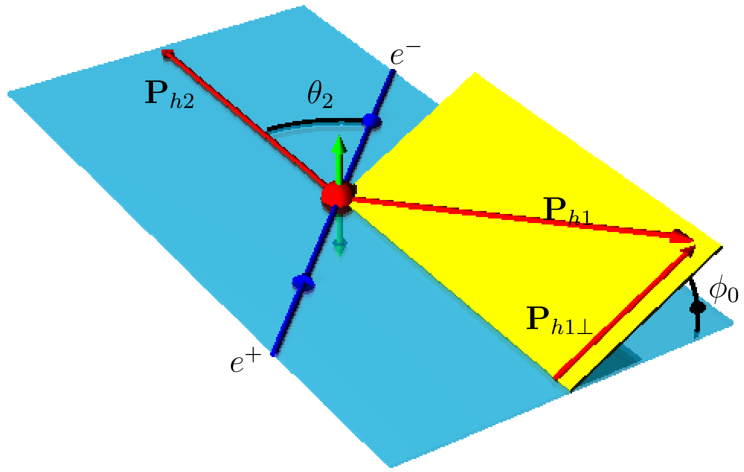
More recently...

$\phi_1+\phi_2$ method



$$\sigma \sim \mathcal{M}_{12}\Big(1 + \frac{\sin^2\theta_T}{1+\cos^2\theta_T}\, cos(\phi_1+\phi_2)\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

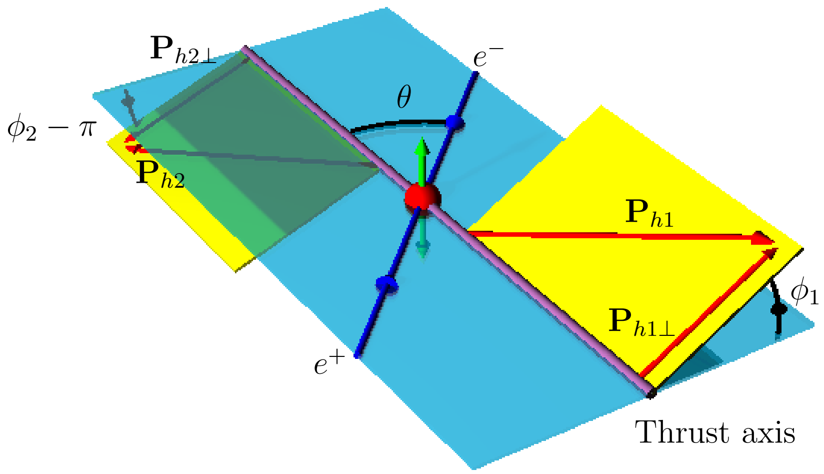
ϕ_0 method



$$\sigma \sim \mathcal{M}_0\Big(1 + \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\, cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$

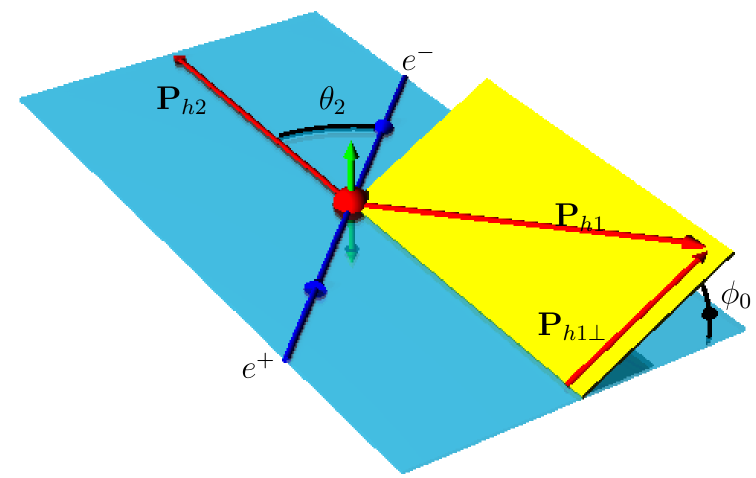
More recently...

$\phi_1+\phi_2$ method

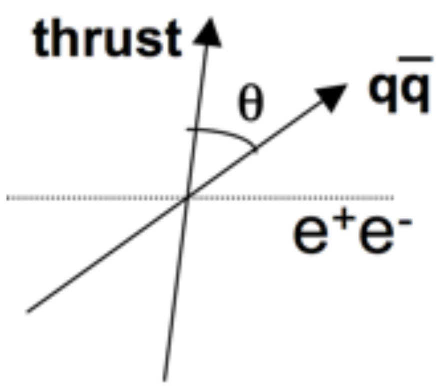


$$\sigma \sim \mathcal{M}_{12}\Big(1+\frac{\sin^2\theta_T}{1+\cos^2\theta_T}\,cos(\phi_1+\phi_2)\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

ϕ_0 method



$$\sigma \sim \mathcal{M}_0\Big(1+\frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$

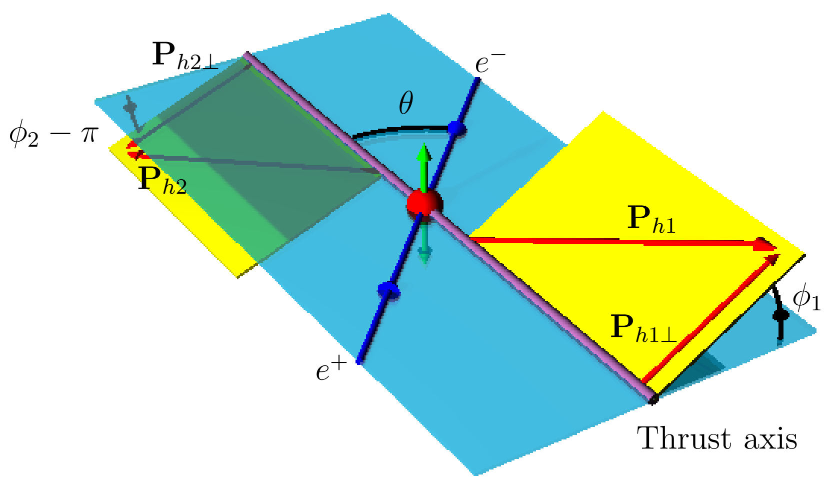


Both interesting: different integration of FFs in p_{Ti}, might provide information on the Collins p_T dependence

Technically more complicated: require the determination of a $q\bar{q}$ proxy (Thrust axis)

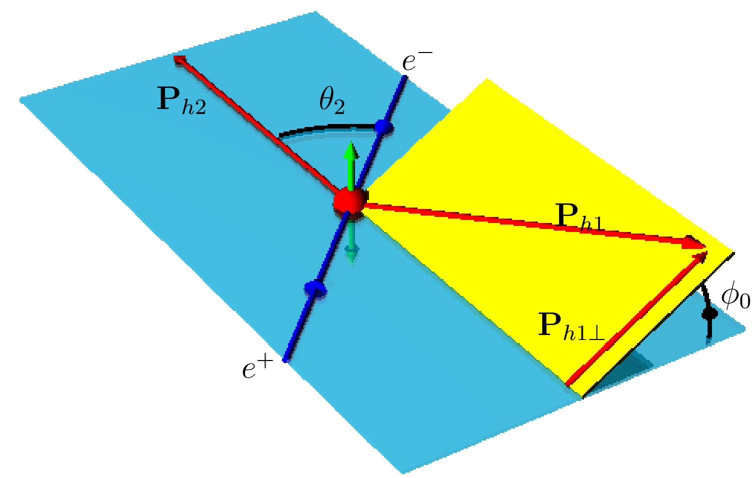
More recently...

$\phi_1+\phi_2$ method

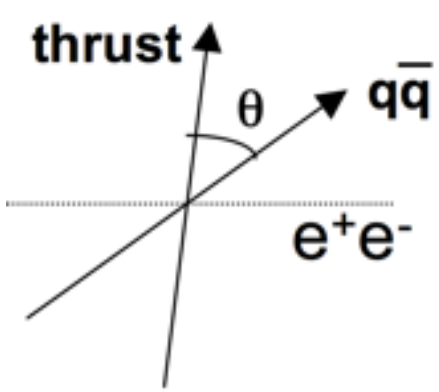


$$\sigma \sim \mathcal{M}_{12}\Big(1+\frac{\sin^2\theta_T}{1+\cos^2\theta_T}\,cos(\phi_1+\phi_2)\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

ϕ_0 method



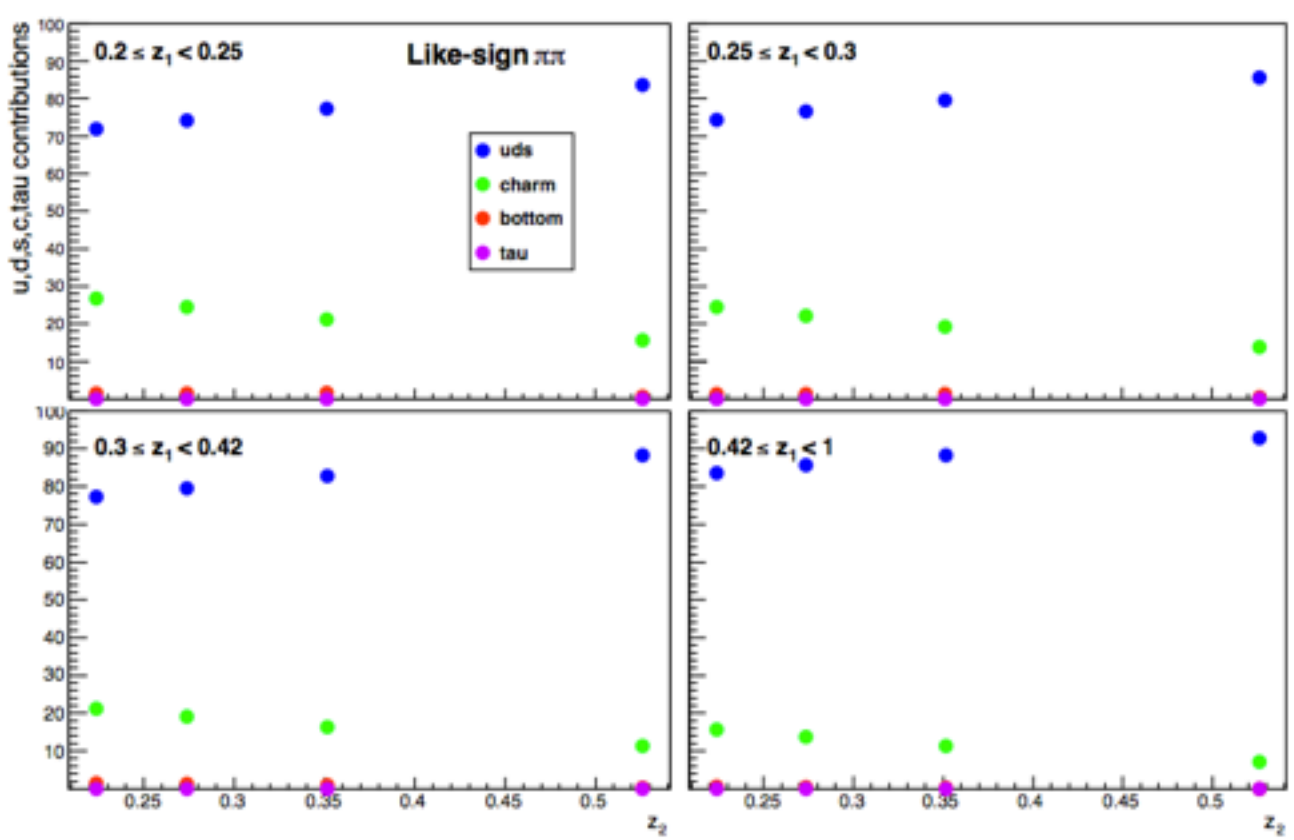
$$\sigma \sim \mathcal{M}_0\Big(1+\frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$



Both interesting: different integration of FFs in p_{Ti}, might provide information on the Collins p_T dependence

Technically more complicated: require the determination of a $q\bar{q}$ proxy (Thrust axis)

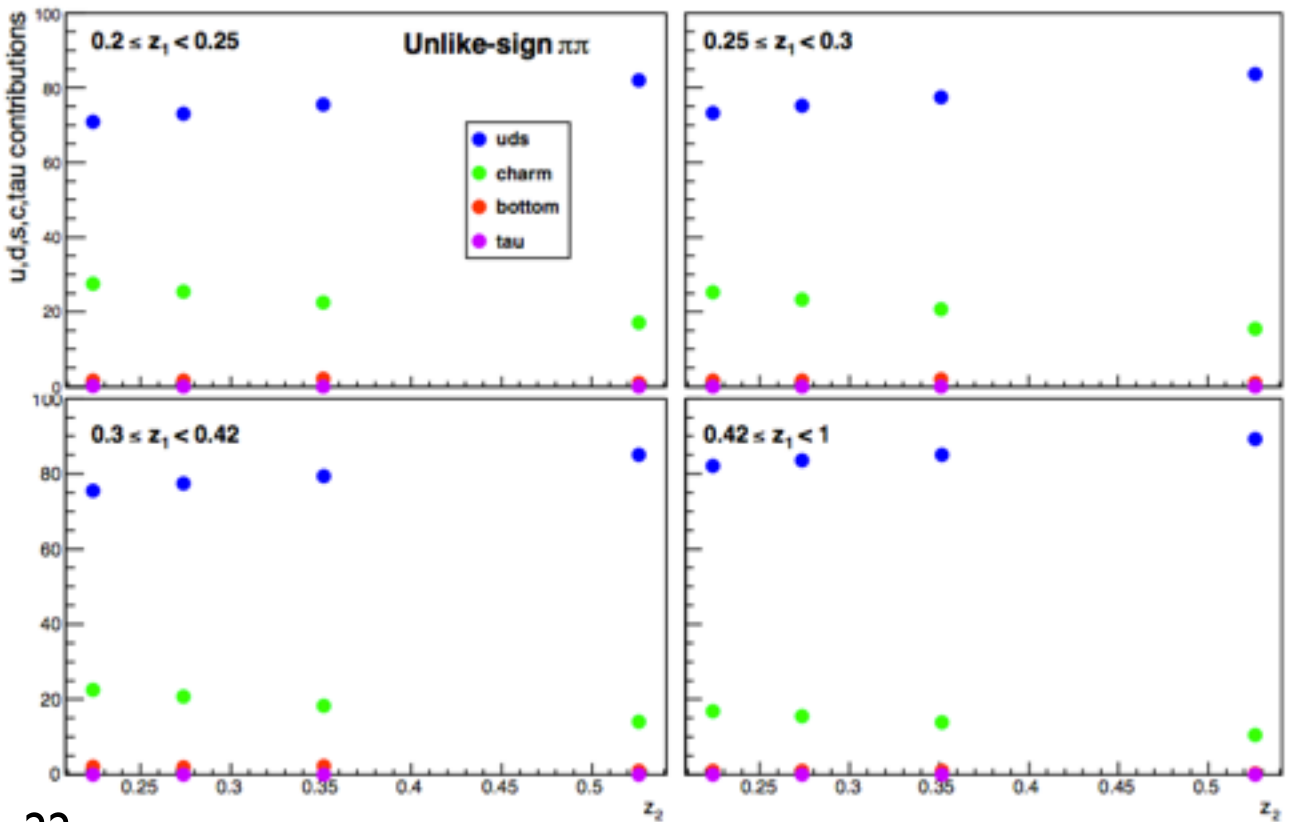
uds-charm-bottom-tau contributions



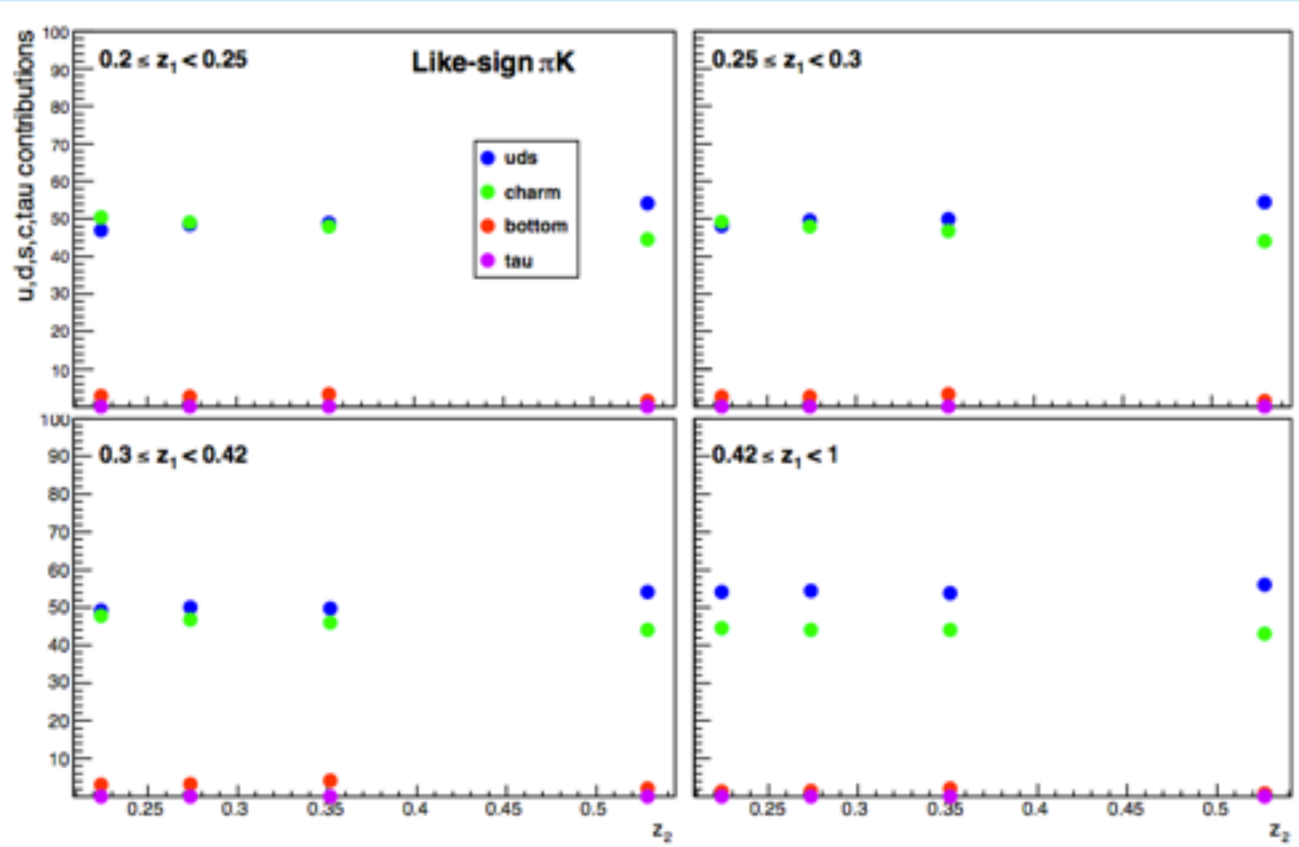
$\pi\pi$ couples

Published $\pi\pi$ studied a charm enhanced data and found charm contribute only as dilution

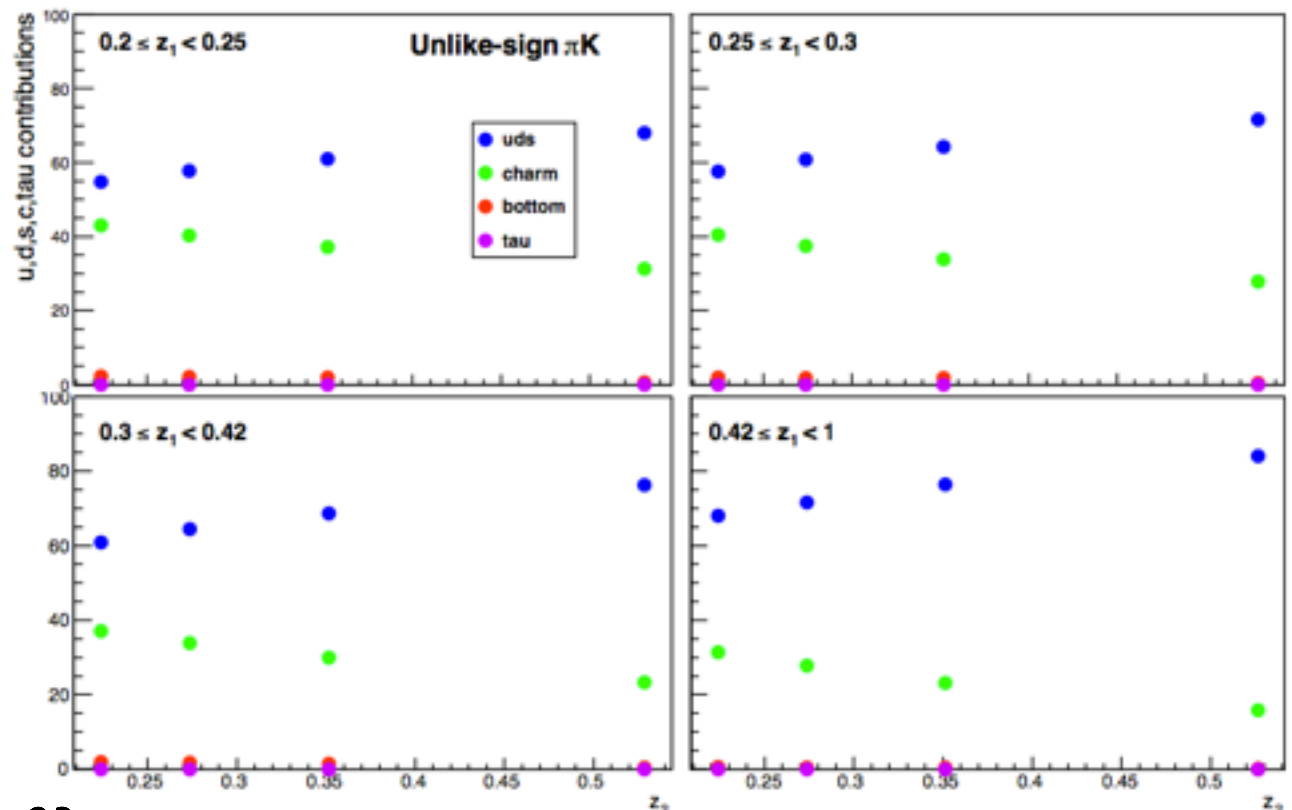
=> charm contribution corrected out



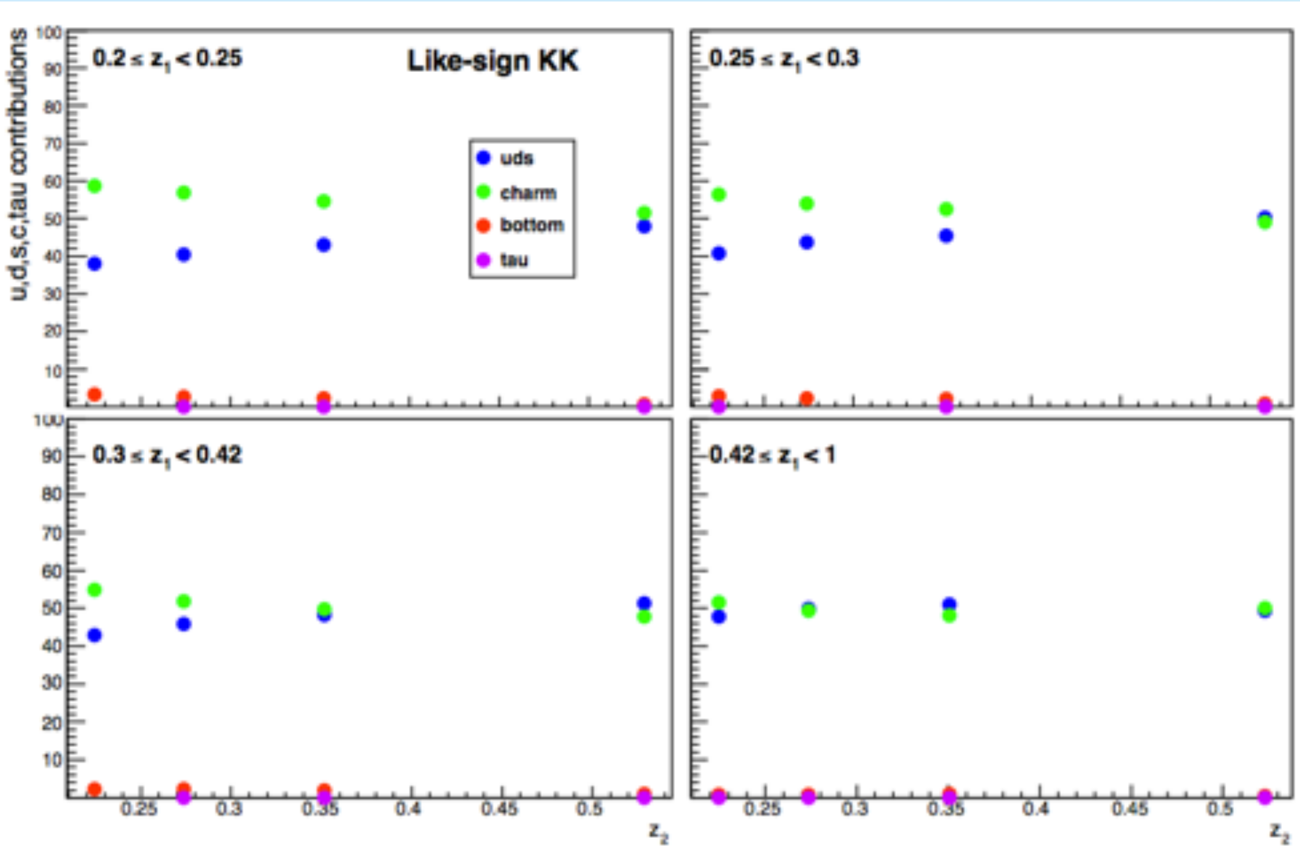
uds-charm-bottom-tau contributions



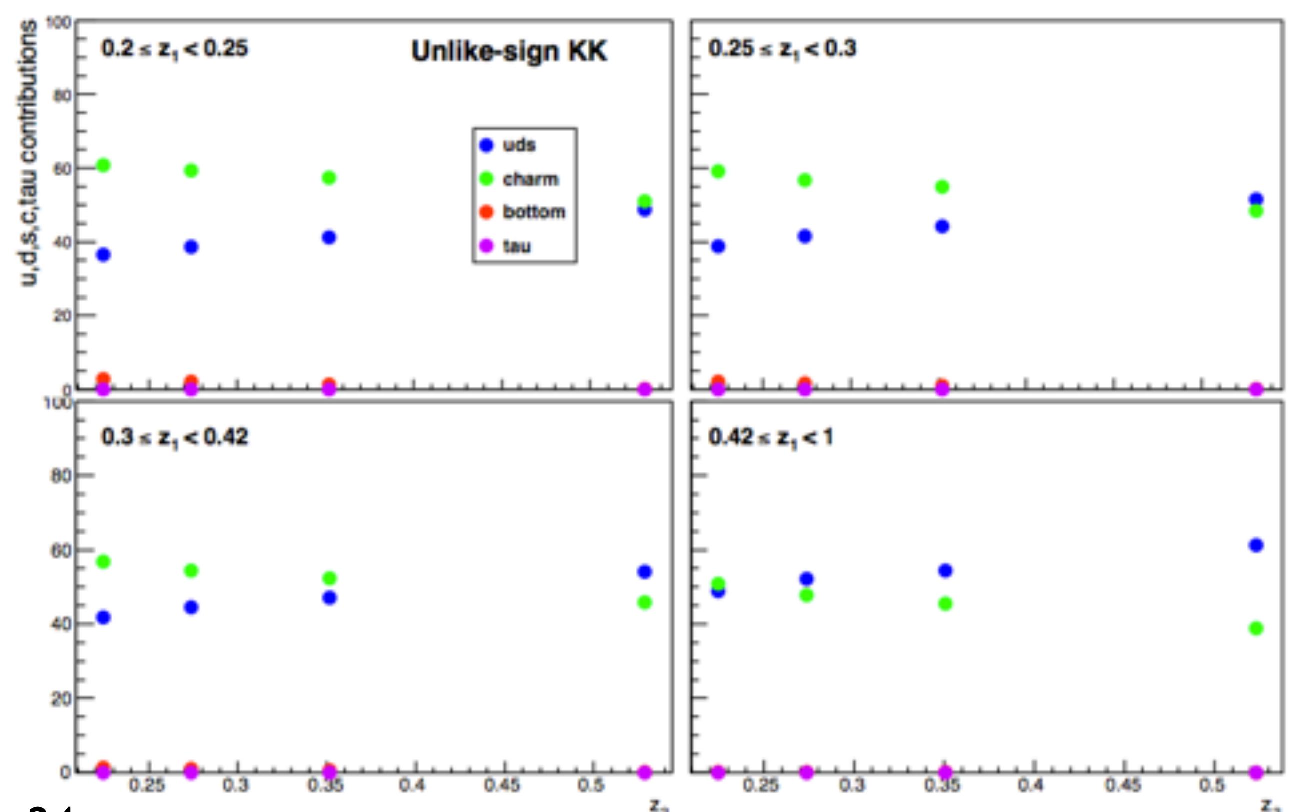
πK couples



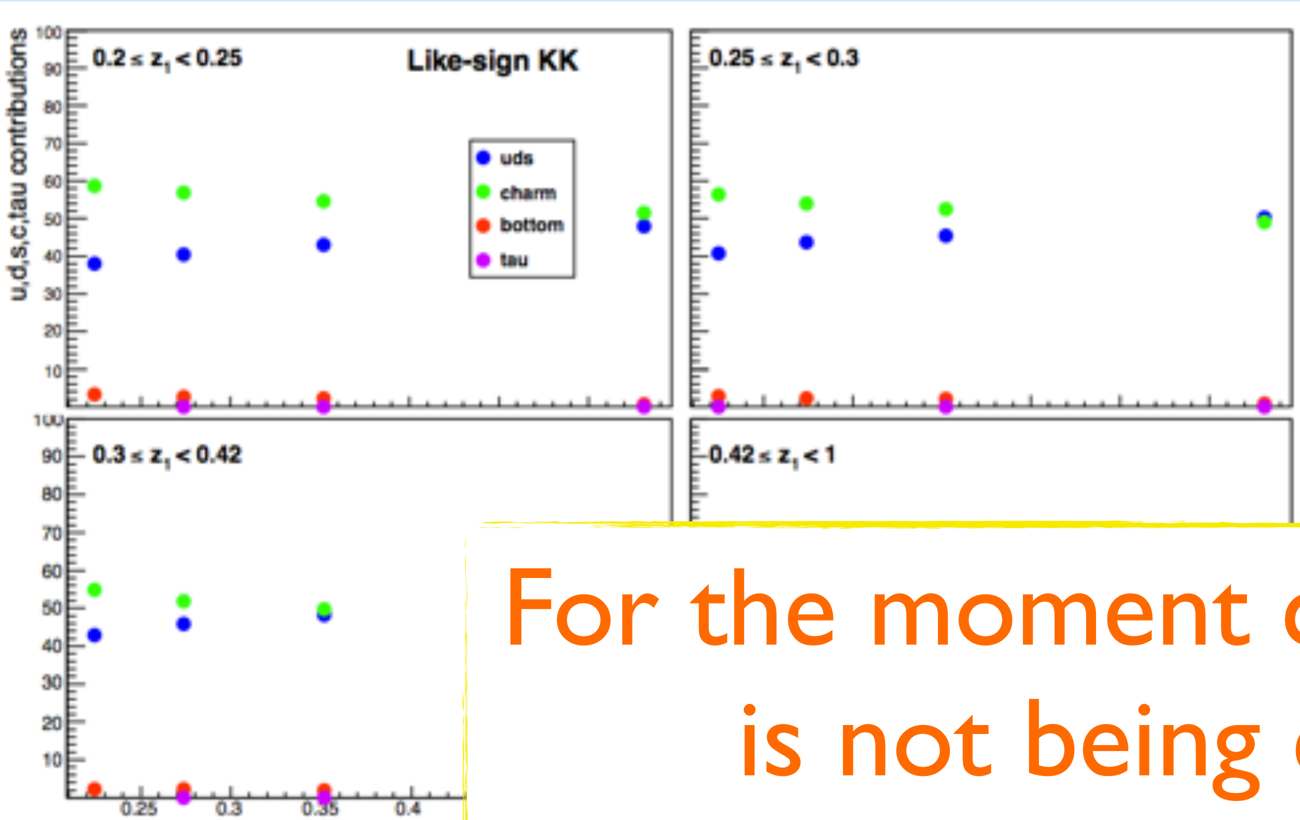
uds-charm-bottom-tau contributions



KK couples

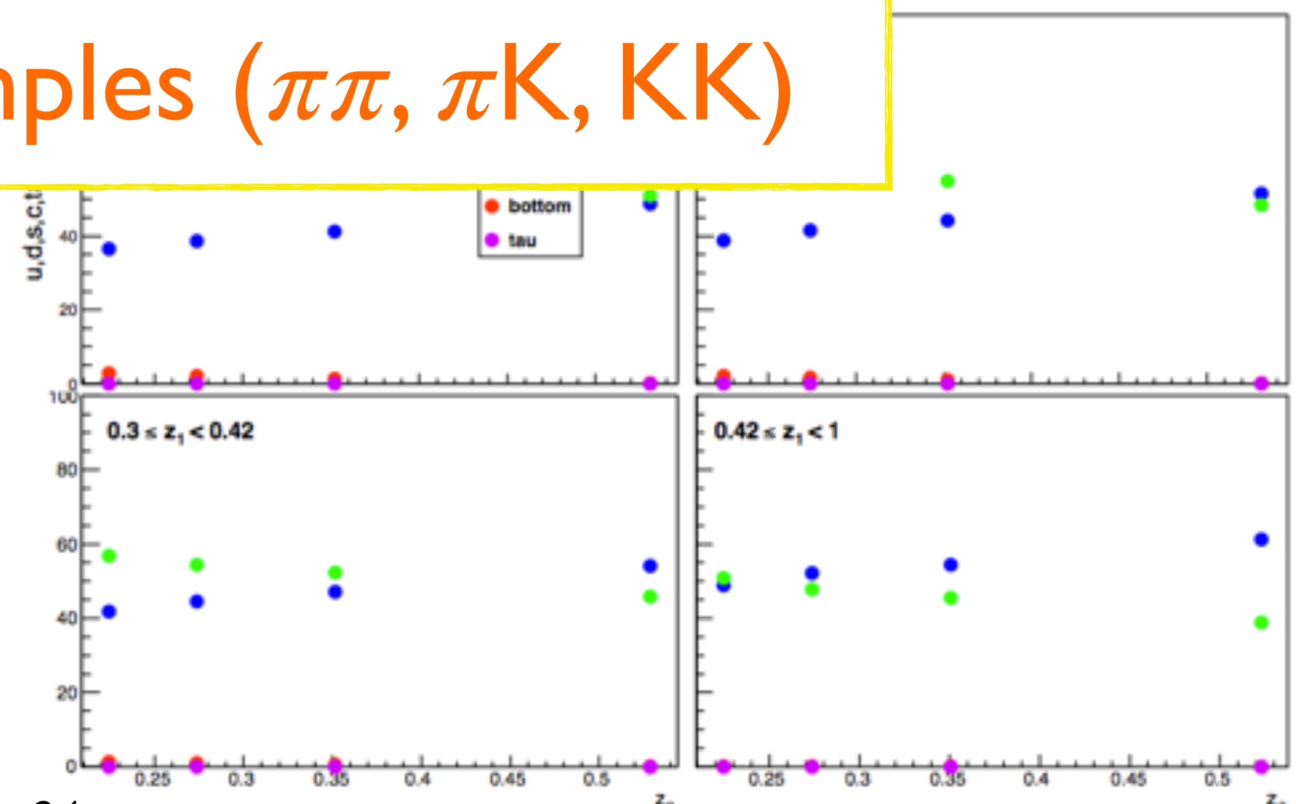


uds-charm-bottom-tau contributions



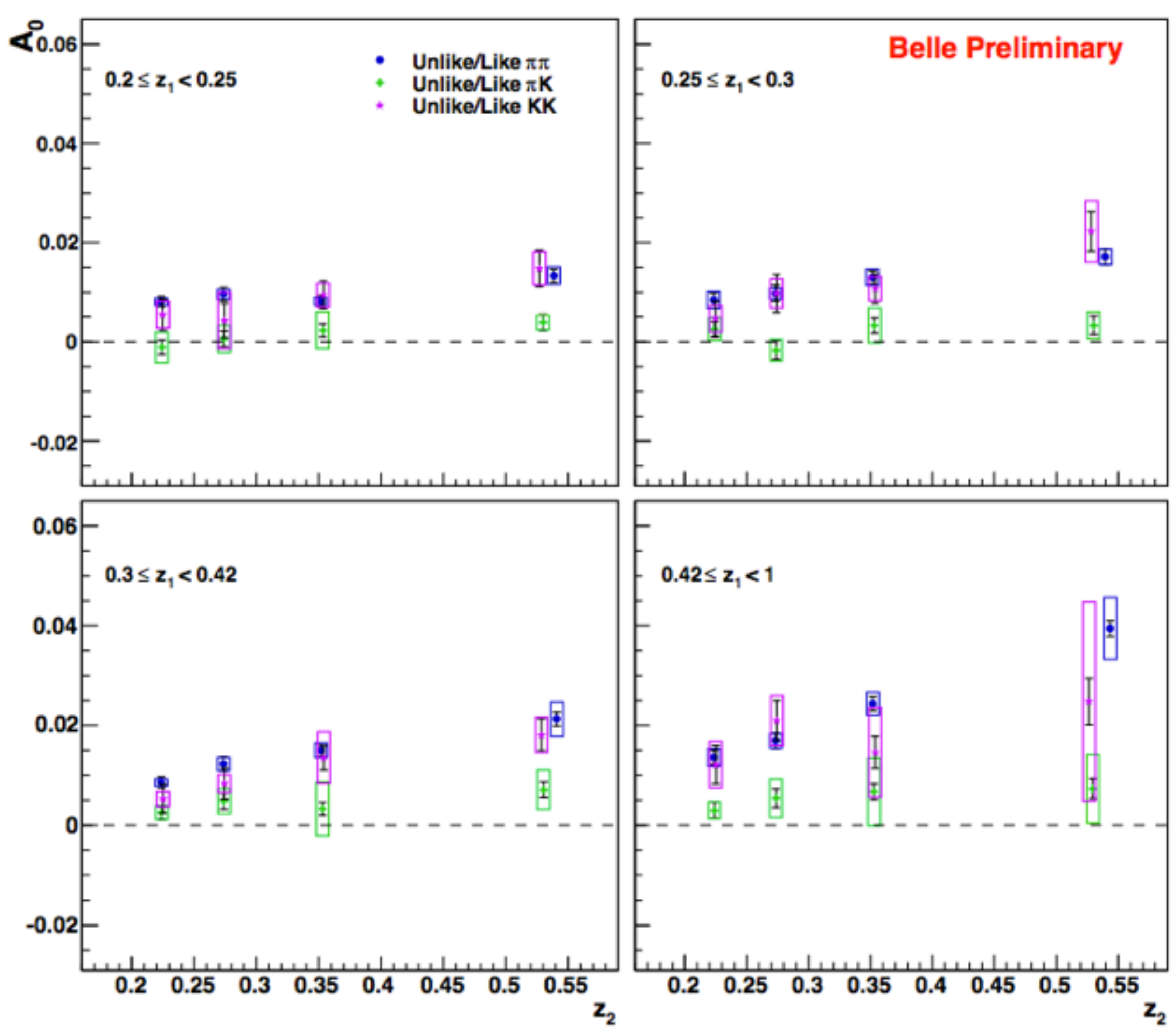
KK couples

For the moment charm contribution is not being corrected out in any of the samples ($\pi\pi$, πK, KK)



Collins asymmetries

ϕ_0 asymmetries

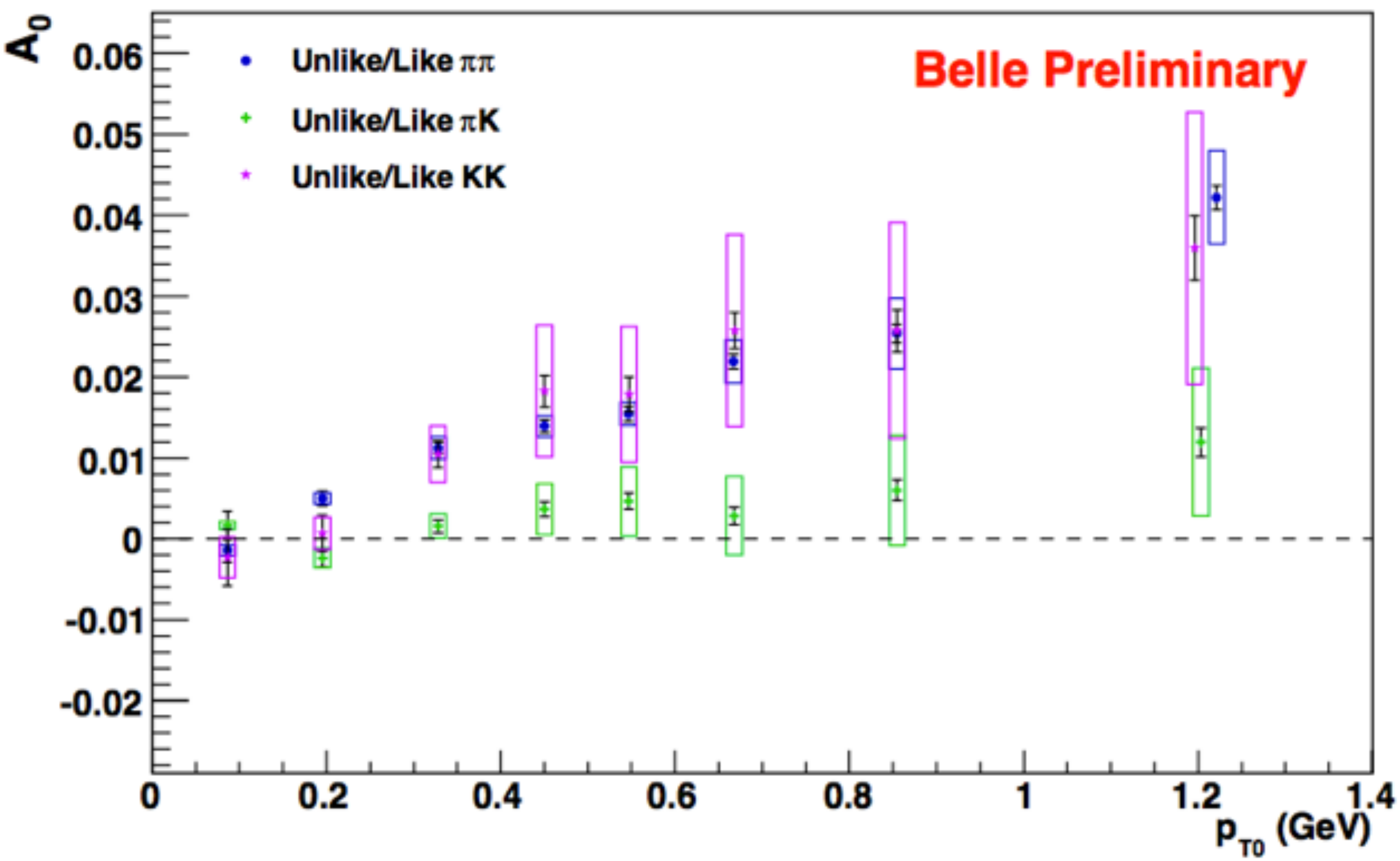


$\pi\pi$ => non-zero asymmetries, increase with z_1, z_2

πK => asymmetries compatible with zero

KK => non-zero asymmetries, increase with z_1,z_2 similar size of pion-pion

ϕ_0 asymmetries



$\pi\pi$ => non-zero asymmetries, increase with z_1, z_2

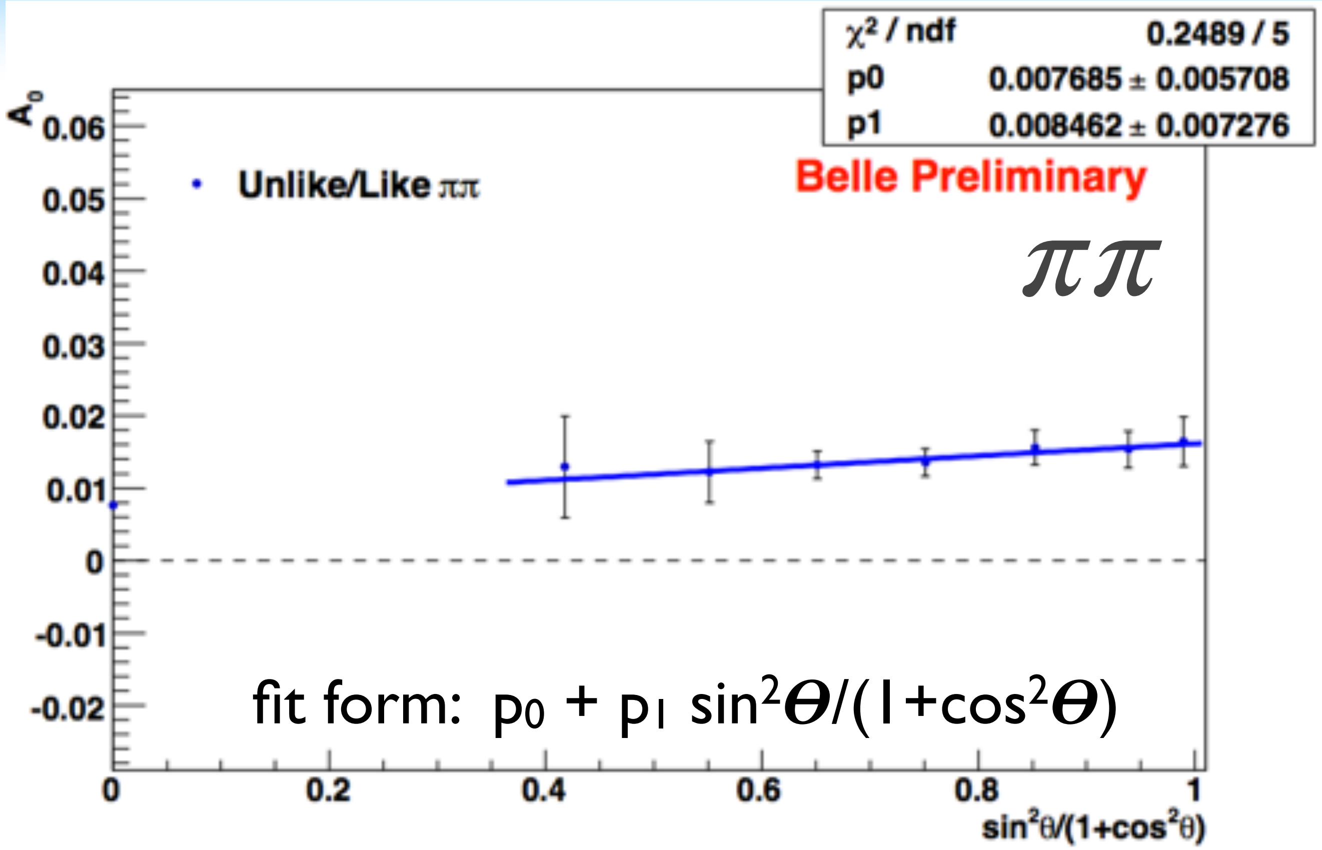
πK => asymmetries compatible with zero

KK => non-zero asymmetries, increase with z_1,z_2 similar size of pion-pion

But! charm have different contributions, we need to account for it!

QCD test?

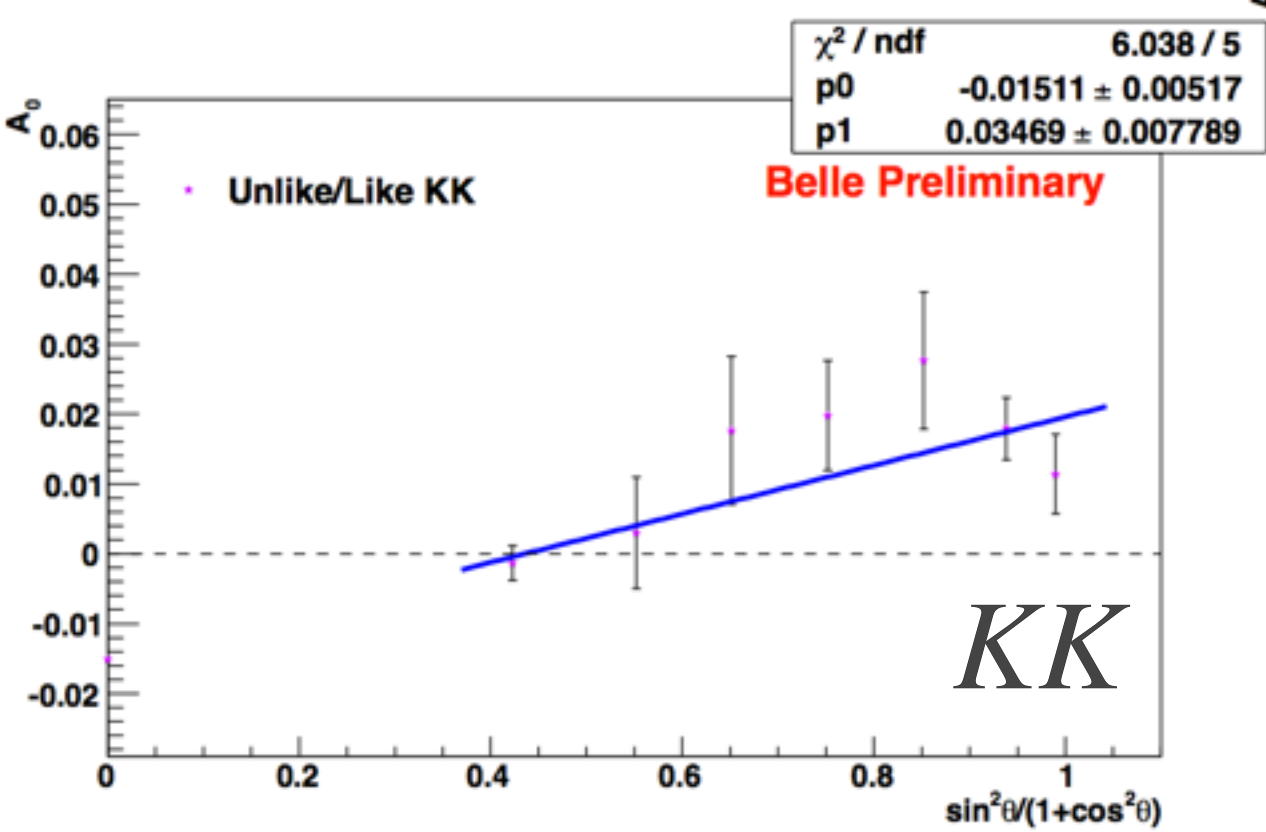
versus $\sin^2\theta/(1+\cos^2\theta)$

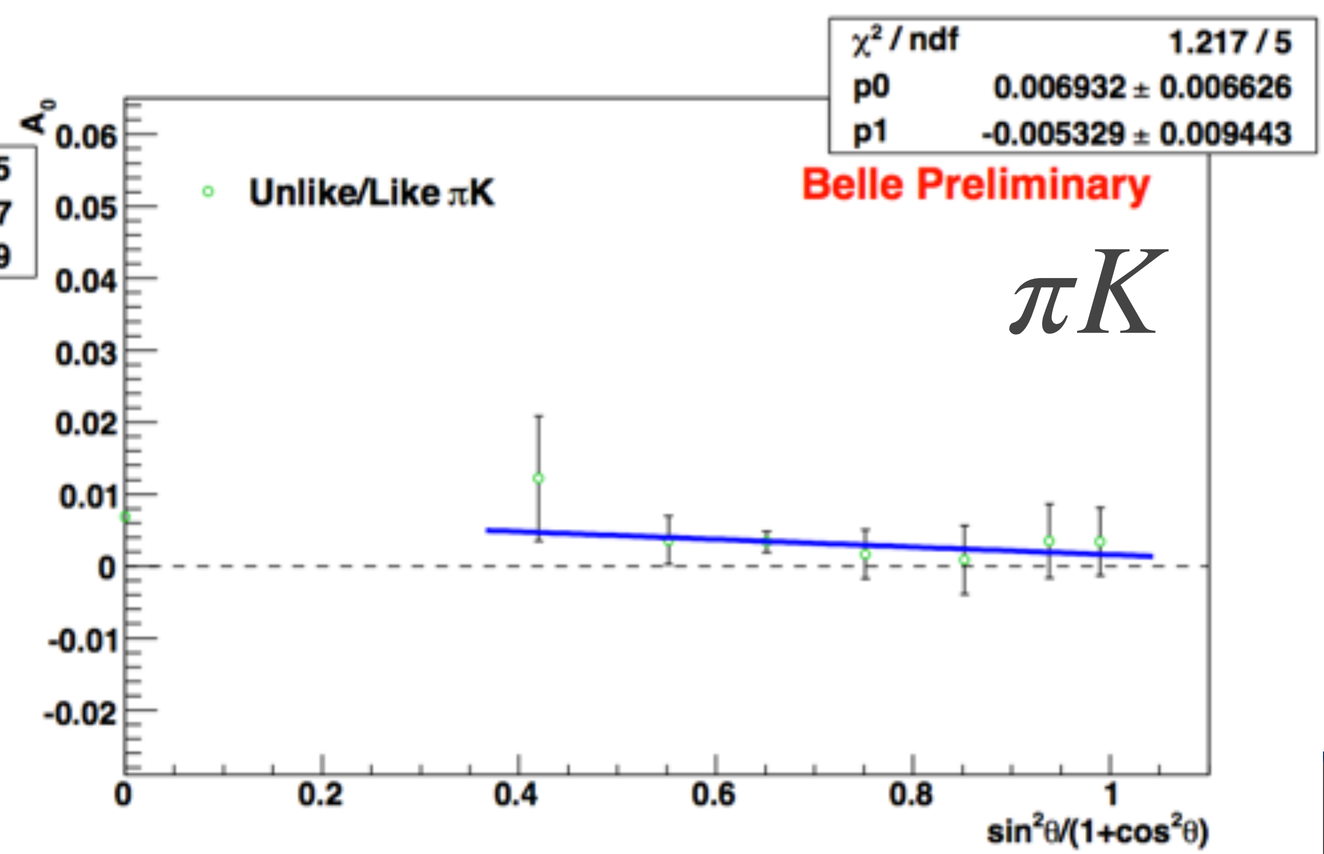


$$A_0 = \frac{\sin^2\theta}{1+\cos^2\theta}\,\mathcal{F}\Big[\frac{H_1^\perp(z_1)\bar{H}_1^\perp(z_2)}{D_1^\perp(z_1)\bar{D}_1^\perp(z_2)}\Big]$$

linear in $\sin^2\theta/(1+\cos^2\theta)$,

go to 0 for $\sin^2\theta/(1+\cos^2\theta) \to 0$





Fragmentation contributions

$$u,\, d \to \pi\ (u\bar{d},\ \bar{u}d)$$

$$D^{fav} = D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+}$$
$$D^{dis} = D_u^{\pi^-} = D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_{\bar{d}}^{\pi^-}$$

$$s \to \pi\ (u\bar{d},\ \bar{u}d)$$

$$D^{dis}_{s\to\pi} = D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-}$$

$$u,\, d \to K\ (u\bar{s},\ \bar{u}s)$$

$$D^{fav}_{u\to K} = D_u^{K^+} = D_{\bar{u}}^{K^-}$$
$$D^{dis}_{u,d\to K} = D_u^{K^-} = D_{\bar{u}}^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^-} = D_d^{K^-} = D_{\bar{d}}^{K^+}$$

$$s \to K\ (u\bar{s},\ \bar{u}s)$$

$$D^{fav}_{s\to K} = D_s^{K^-} = D_{\bar{s}}^{K^+}$$
$$D^{dis}_{s\to K} = D_s^{K^+} = D_{\bar{s}}^{K^-}$$

In the end we are left with 7 possible fragmentation functions:

$$D^{fav},\ D^{dis},\ D^{dis}_{s\to\pi},\ D^{fav}_{u\to K},\ D^{dis}_{u,d\to K},\ D^{fav}_{s\to K},\ D^{dis}_{s\to K}$$

Assuming charm contribute only as a dilution

Fragmentation contributions

For pion-pion couples:

$$D^{\frac{U_{\pi\pi}}{L_{\pi\pi}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \left(\frac{5H_1^{fav} H_2^{fav} + 5H_1^{dis} H_2^{dis} + 2H^{dis}_{1\,s\to\pi} H^{dis}_{2\,s\to\pi}}{5D_1^{fav} D_2^{fav} + 5D_1^{dis} D_2^{dis} + 2D^{dis}_{1\,s\to\pi} D^{dis}_{2\,s\to\pi}} - \frac{5H_1^{fav} H_2^{dis} + 5H_1^{dis} H_2^{fav} + 2H^{dis}_{1\,s\to\pi} H^{dis}_{2\,s\to\pi}}{5D_1^{fav} D_2^{dis} + 5D_1^{dis} D_2^{fav} + 2D^{dis}_{1\,s\to\pi} D^{dis}_{2\,s\to\pi}} \right)$$

For pion-Kaon couples:

$$D^{\frac{U_{\pi K}}{L_{\pi K}}} \propto 1 + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \times$$

$$\left(\frac{4H_1^{fav} H_{2\,K}^{fav} + H_{1K}^{dis}(5H_2^{dis} + H_2^{fav}) + H_{2K}^{dis}(5H_1^{dis} + H_1^{fav}) + 4\,H_{1K}^{fav} H_2^{fav} + H^{dis}_{1s\to\pi}(H^{dis}_{2s\to K} + H^{fav}_{2s\to K}) + H^{dis}_{2s\to\pi}(H^{fav}_{1s\to K} + H^{dis}_{1s\to K})}{4D_1^{fav} D_{2\,K}^{fav} + D_{1K}^{dis}(5D_2^{dis} + D_2^{fav}) + D_{2K}^{dis}(5D_1^{dis} + D_1^{fav}) + 4\,D_{1K}^{fav} D_2^{fav} + D^{dis}_{1s\to\pi}(D^{dis}_{2s\to K} + D^{fav}_{2s\to K}) + D^{dis}_{2s\to\pi}(D^{fav}_{1s\to K} + D^{dis}_{1s\to K})} \right.$$

$$\left. \frac{H_{2\,K}^{dis}(5H_1^{fav} + H_1^{dis}) + 4\,H_{1\,K}^{fav} H_2^{dis} + 4H_1^{dis} H_{2K}^{fav} + H_{1K}^{dis}(5H_2^{fav} + H_2^{dis}) + H^{dis}_{1s\to\pi}(H^{fav}_{2s\to K} + H^{dis}_{2s\to K}) + (H^{dis}_{1s\to K} + H^{fav}_{1s\to K})H^{dis}_{2s\to\pi}}{D_{2\,K}^{dis}(5D_1^{fav} + D_1^{dis}) + 4\,D_{1\,K}^{fav} D_2^{dis} + 4D_1^{dis} D_{2K}^{fav} + D_{1K}^{dis}(5D_2^{fav} + D_2^{dis}) + D^{dis}_{1s\to\pi}(D^{fav}_{2s\to K} + D^{dis}_{2s\to K}) + (D^{dis}_{1s\to K} + D^{fav}_{1s\to K})D^{dis}_{2s\to\pi}} \right)$$

For Kaon-Kaon couples:

$$D^{\frac{U_{KK}}{L_{KK}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \left(\frac{4H_{1K}^{fav} H_{2\,K}^{fav} + 6H_{1\,K}^{dis} H_{2K}^{dis} + H^{dis}_{1s\to K} H^{dis}_{2s\to K} + H^{fav}_{1s\to K} H^{fav}_{2s\to K}}{4D_{1K}^{fav} D_{2\,K}^{fav} + 6D_{1\,K}^{dis} D_{2K}^{dis} + D^{dis}_{1s\to K} D^{dis}_{2s\to K} + D^{fav}_{1s\to K} D^{fav}_{2s\to K}} - \frac{4H_{1K}^{fav} H_{2\,K}^{dis} + 4H_{1\,K}^{dis} H_{2K}^{fav} + 2H_{1K}^{dis} H_{2K}^{dis} + H^{dis}_{1s\to K} H^{fav}_{2s\to K} + H^{fav}_{1s\to K} H^{dis}_{2s\to K}}{4D_{1K}^{fav} D_{2\,K}^{dis} + 4D_{1\,K}^{dis} D_{2K}^{fav} + 2D_{1K}^{dis} D_{2K}^{dis} + D^{dis}_{1s\to K} D^{fav}_{2s\to K} + D^{fav}_{1s\to K} D^{dis}_{2s\to K}} \right)$$

Fragmentation contributions

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$$D^{\frac{U_{\pi K}}{L_{\pi K}}} \propto 1 + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \times$$

$$\left(\frac{4H_1^{fav} H_{2\,K}^{fav} + H_{1K}^{dis}(5H_2^{dis} + H_2^{fav}) + H_{2K}^{dis}(5H_1^{dis} + H_1^{fav}) + 4\,H_{1K}^{fav} H_2^{fav} + H^{dis}_{1s\to\pi}(H^{dis}_{2s\to K} + H^{fav}_{2s\to K}) + H^{dis}_{2s\to\pi}(H^{fav}_{1s\to K} + H^{dis}_{1s\to K})}{4D_1^{fav} D_{2\,K}^{fav} + D_{1K}^{dis}(5D_2^{dis} + D_2^{fav}) + D_{2K}^{dis}(5D_1^{dis} + D_1^{fav}) + 4\,D_{1K}^{fav} D_2^{fav} + D^{dis}_{1s\to\pi}(D^{dis}_{2s\to K} + D^{fav}_{2s\to K}) + D^{dis}_{2s\to\pi}(D^{fav}_{1s\to K} + D^{dis}_{1s\to K})} \right.$$

$$\left. \frac{H_{2\,K}^{dis}(5H_1^{fav} + H_1^{dis}) + 4\,H_{1\,K}^{fav} H_2^{dis} + 4H_1^{dis} H_{2K}^{fav} + H_{1K}^{dis}(5H_2^{fav} + H_2^{dis}) + H^{dis}_{1s\to\pi}(H^{fav}_{2s\to K} + H^{dis}_{2s\to K}) + (H^{dis}_{1s\to K} + H^{fav}_{1s\to K})H^{dis}_{2s\to\pi}}{D_{2\,K}^{dis}(5D_1^{fav} + D_1^{dis}) + 4\,D_{1\,K}^{fav} D_2^{dis} + 4D_1^{dis} D_{2K}^{fav} + D_{1K}^{dis}(5D_2^{fav} + D_2^{dis}) + D^{dis}_{1s\to\pi}(D^{fav}_{2s\to K} + D^{dis}_{2s\to K}) + (D^{dis}_{1s\to K} + D^{fav}_{1s\to K})D^{dis}_{2s\to\pi}} \right)$$

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Not so easy! A full phenomenological study needed!